

# Dynamic Analysis Of A Snap-Action Micromechanism

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## Abstract

Snap-action bistable mechanisms have a practical use in applications when very few but well defined and repeatable states of a micro-mechanism are required. Some obvious applications include micro-switches, addressable MEMS-based pixel arrays, and tunable optical filters. We have explored the dynamic switching phenomena analytically and experimentally. This paper reports on a detailed mathematical model governing the non-linear response of a bistable micromechanism in its bistable equilibrium, analytical results describing non-linear dynamics of the mechanism near its equilibrium state and transient dynamics of switching between bistable states, and design, fabrication, and characterization results bistable micromechanism in their upper bistable equilibrium.

## Keywords

Bistable micromechanism, MEMS, dynamic switching.

## INTRODUCTION

An advantage of snap-action mechanisms is that no power is required to keep the mechanism in either of its bistable states; structural non-linearity typically provides state's static equilibrium. Conventionally, the switching between bistable states is static, utilizing either static [1] or thermal actuation [2].

Several snap-action behavior micromechanisms have already been introduced and behavior of these mechanisms have been carefully analyzed [3]. All snap-action mechanisms have two stable states and when predefined perturbation (critical force or moment) is applied a fast non-linear transition from one state to the other happens. Energy is only needed to switch between states, but no energy is needed to keep devices in equilibrium state. Different examples of microbistable systems have been presented [4],[5] and recently these properties have been used to built several micro-devices such as memory cells [6], microswitches [7], micro relays [8], microvalves [9] and fiber optic switches [10]. All these mechanisms were actuated statically, using thermal or electrostatic actuators. In order to understand the behavior of the bistable mechanism and improve its design, there had been several authors studying carefully how to use bistability, mainly focusing on static loads to switch between states [11],[12].

In contrast to the conventional bistable microswitches utilizing static actuation, we propose to analyze the design approach utilizing an electrostatic actuation system which capable to switch dynamically. The goal of using dynamic

forces is to reduce power consumption by driving the device into resonance. At the resonance frequency the amplitude of motion is maximized for the same energy input.

In this paper, the mechanism we have considered to achieve bistability is a truss-like mechanism based on two slightly bended beams, Figure 1. To allow the rotation of the beam from state to state we substitute hinges by flexural hinges, allowing the mechanism to be micromachined the other state there is a need for hinge which are substituted by flexural hinges so the mechanism can be micromachined. The micromechanism is also integrating two lateral springs at the end of the beam to relieve stress during switching.

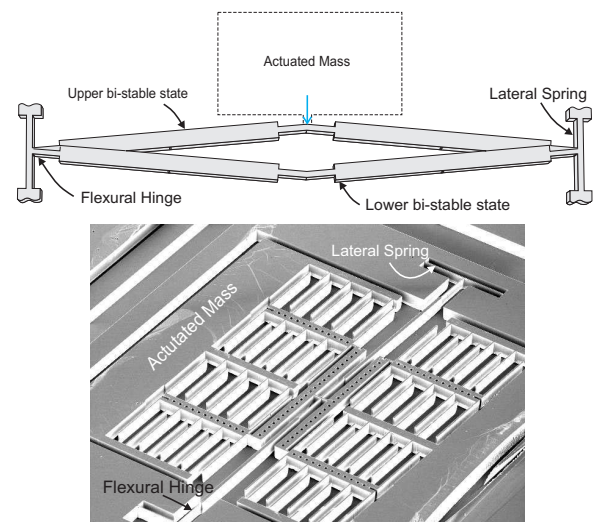


Figure 1. Compliant bistable snap-action mechanism.

## PHENOMENON

This paper explores a paradigm utilizing structural resonance phenomena to switch dynamically between bistable states. If a linear structure is driven into resonance (the frequency of the excitation force is equal to the natural frequency of the structure), the structure can achieve a relatively large amplitude of oscillation using a relatively small absolute amplitude actuation force. In context of a bistable mechanism, under certain conditions, the structure driven into resonance-like vibration may achieve a large enough deviation from its equilibrium, sufficient to switch between bistable states. In Figure 2, three switching strategies of a bistable mechanism are illustrated - one static and two dynamic. Only the dynamic resonance case provides power saving advantages.

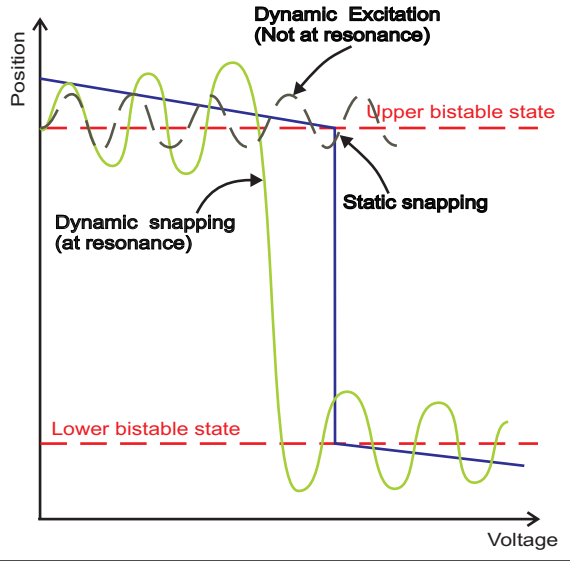


Figure 2. Static and dynamic switching strategies.

## DESIGN

A truss like-micromechanism such as the one in Figure 3, is an example of a nonlinear system where the nonlinearity comes from its geometry. In this system, when an applied force is negative, the structural stiffness decreases and indeed approaches to zero (unstable state), after this stage the micromechanism "snaps" to the next equilibrium point. This micromechanism is formed by two beams inclined  $\theta_0$  degrees that are free to rotate. The rotation is achieved through flexural hinges of width approximately 10 % of the beam's width and 10 % of the beam's length. To relieve the stress when switching two stress-relieve beams are placed at the ends of the beams. The mechanism is thought to be mi-

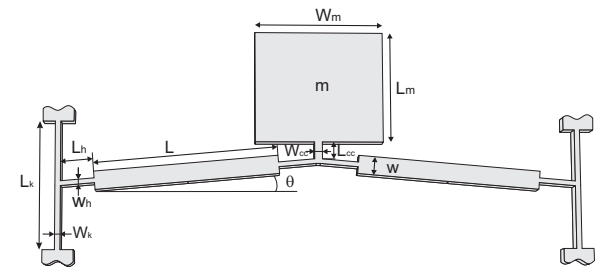


Figure 3. Design of the compliant bistable snap-action mechanism.

cromachined, in this case using a SOI bulk micromachining process. The parameters chosen to implement the device are in Table 1.

In the proposed design concept, the mass is electrostatically forced to oscillate to the resonance frequency of the system in the drive direction. The actuators are comb drive actuators, applying the harmonic force at the central part of the beam.

Table 1. Dimensions of the bistable micromechanism.

Dimensions	Description	Value
$L_k$	Length of the lateral beam	$40\mu m$
$w_k$	Width of the lateral beam	$7\mu m$
$L_h$	Length of flexures	$100\mu m$
$w_h$	Width of flexures	$7\mu m$
$L$	Beam length	$1050.06\mu m$
$w$	Beam width	$70\mu m$
$L_{cc}$	Length of the central beam	$24\mu m$
$w_{cc}$	Width of the central beam	$14\mu m$
$L_m$	Mass Length	$2000\mu m$
$w_m$	Mass width	$2000\mu m$
$\tau$	Thickness	$100\mu m$
$\theta_0$	Initial angle	$0.55^\circ$

The electrostatic force generated in the y-direction as comb drives slide over each other can be approximated as:

$$F_e = \frac{\epsilon_0 V^2 z_0}{2y_0} \quad (1)$$

Where  $\epsilon_0 = 8.854 \times 10^{-12} F/m$ ,  $V$  is the voltage applied, with a driving frequency equal to the natural frequency of the system,  $z_0$  is the thickness of the comb drive, and  $y_0$  is the gap between combs.

## MODEL

Applying the theory of the pseudo-rigid body model [14] to the compliant mechanism described in Figure 3, a model has been obtained, Figure 4. Mass  $m$  of the whole structure is assumed to be concentrated in the point B, the lateral beams with length and width ( $L_k, W_k$ ) are modeled as linear springs of spring constant  $k_1$ , calculated using the clamped-clamped spring constant of a cantilever beam ( $k_1 = \frac{192EI}{L_k^3}$ ). The flexures have been modeled as torsional springs with the spring constant  $k = \frac{Et w_h^3}{12L_h}$ , (see [13] for details).

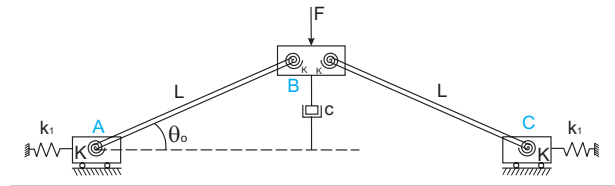


Figure 4. Model of the compliant bistable snap-action mechanism.

Ignoring the mass of the micromechanism itself and considering all the mass concentrated in B, the dynamic equilibrium of the large concentrated mass can be written as the equation of motion in the form of a single degree-of-freedom oscillator:

$$m \frac{d^2 \Delta y}{dt^2} + c \frac{d \Delta y}{dt} + F_m = F_e \quad (2)$$

where  $\Delta y = L \sin \theta - L \sin \theta_0$  is the vertical displacement of the concentrated mass from the upper bistable level,  $m$  is the concentrated mass,  $F_m$  is the structural stiffness of the micromechanism, that can be calculated applying equilibrium conditions to the beam and then obtaining the relation between the force applied and the displacement of the mass:

$$F_m = 2k_1 L^2 (\cos(\theta) - \cos(\theta_0)) \tan(\theta) + \frac{4k(\theta_0 - \theta)}{L \cos(\theta)} \quad (3)$$

The  $F_e$  is the electrostatic force externally applied, described in Eq. 1 where  $V$  is a sinusoidal voltage. Parameter  $c$  is the damping coefficient which is mainly due to the viscous effects of the air between the mass and the substrate, and in between the comb-drive capacitor fingers. These viscous damping effects can be captured by using Couette flow damping, the total damping can be expressed as

$$c = \mu_p p \frac{A}{z_0} + \mu_p p \frac{2Nl_{comb}t}{y_{comb}} \quad (4)$$

where  $A$  is the area of the mass,  $z_0$  is the elevation of the proof mass from the substrate,  $t$  is the thickness of the structure,  $N_{comb}$  is the number of comb-drive fingers,  $y_{comb}$  is the distance between the fingers,  $l_{comb}$  is the length of the fingers,  $p$  is the ambient pressure within the cavity of the packaged device, and  $\mu_p$  is the viscosity constant for air.

## RESULTS

### Static actuation

The micromechanism analyzed can be considered as a displacement driven problem, see Figure 5.  $A$  is the upper stable state of the system but if  $y$  decreases, the microsystem reaches the state  $B$ . At this point the maximum force is needed after the system becomes unstable  $C$  and the snap-through phenomenon takes place. At this point the system makes a large displacement to the next equilibrium position. A similar transition will happen when increasing distance from the second lower stable state  $D$ . The system would follow the curve till the unstable point and snap-through the upper stable point. It can also be noticed in Figure 5 that the system is non-symmetric; this is due to flexures. The torque caused by the flexures generate the asymmetry and more energy is needed to switch from the upper stable state to the lower, more than from the lower to the upper states. Finite element analysis has been used to verify the analytical model. In Figure 5 the static behavior of the FEA model is plotted, matching closely the analytic behavior described previously. The minimum static perturbation voltage (aprox. 24V) was obtained applying a ramp force to the analytical model.

### Dynamic actuation

The same micromechanism was now driven by an electrostatic force with a sinusoidal voltage applied at different frequencies. When the frequency of the external force was

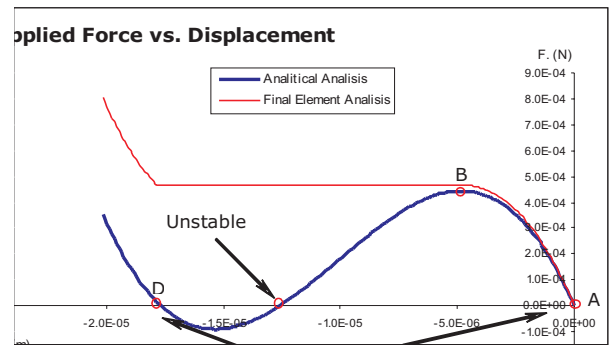


Figure 5. Load -deflection curve.

at the linear resonance frequency of the system, the system accumulates enough energy to snap to the other state, see in Figure 6 at a voltage lower than the static snapping voltage.

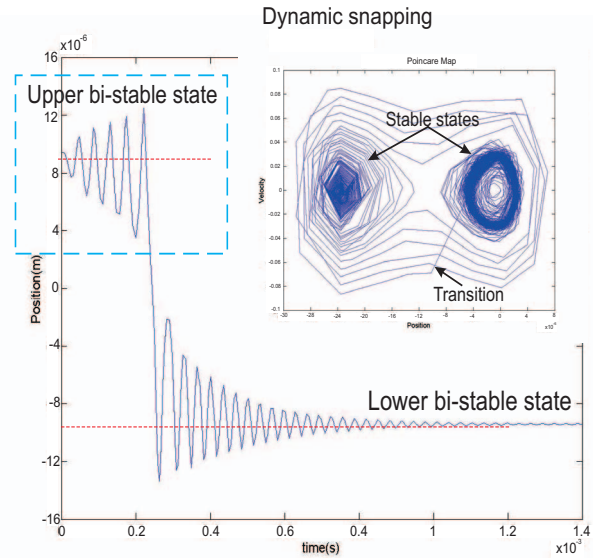


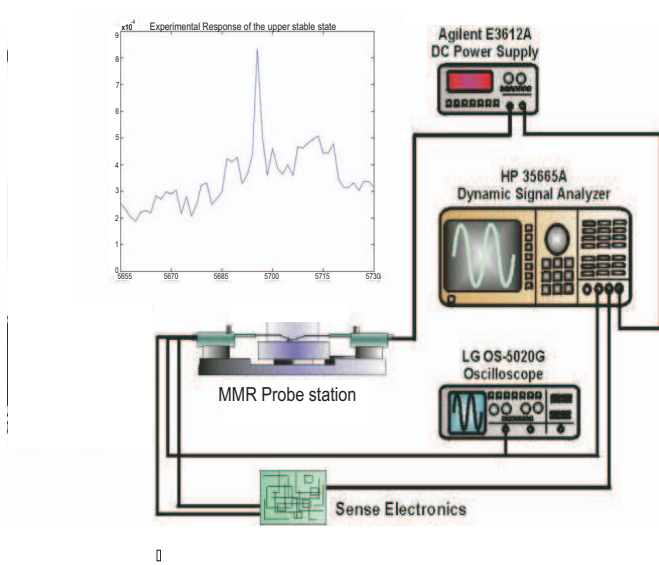
Figure 6. Dynamic Snapping when driven at  $15 V_{pp}$ , at frequency=  $\omega_n$  vs. frequency=  $2\omega_n$  at the same driving voltage

In the Figure 6, the system is driven by a comb drive actuator with a sinusoidal voltage applied with 15 V pick-to pick. When the frequency is the natural frequency of the system snapping occurs while when driven at different frequencies the system just oscillates around the upper stable state.

## Experimental

We designed, fabricated, and characterized bistable test structures. The devices are fabricated using an SOI-based process developed by the group. In Figure 1, an SEM of the device is shown. The inclined beams are  $2500 \mu m$  long,  $100 \mu m$  thick,  $10 \mu m$  wide and initial apex of the buckled beam of

12  $\mu\text{m}$ . The bistable test structures are actuated electrostatically using parallel plate electrodes on each side of the microstructure.



**Figure 7. Testing set-up**

The structures were biased with  $V_{DC} = 25V$  and driven by  $V_{AC} = 3.53V$ . The structure was characterized in a vacuum chamber under 200mTorr pressure, see Figure 7. The response of devices was detected both electrostatically and optically. The experimental measurements demonstrated the non-linear behavior of the resonators with the central frequency near 5.7kHz, closely matching analytical and FEA results.

## CONCLUSIONS

In this paper, a new strategy is explored to switch between states in a bistable structure, utilizing structural resonance phenomena. In order to test this strategy, a truss-like micromechanism with a nonlinear geometry has been designed. An analytical model of the system has been derived using pseudo-rigid body model. The feasibility of driving the micromechanism into resonance and achieving dynamical switching has been illustrated. This strategy reduces the power consumption for a micromechanism of the dimensions specified, because when the system is driven dynamically (with a sinusoidal voltage using a comb drive actuator only) only  $V_{AC} = 15V$  are needed to switch, whereas when driven statically (thermal or electrostatic actuation) higher voltages in the order of 25V. The analytical model has been verified using FEA modeling and the static behavior has been proven to be the same, and also the resonance frequency of the system when driving the system dynamically. Several tests structures of the mechanism were fabricated using an SOI-based process developed by the group, and the behavior

near the equilibrium was tested and agree with the analytical behavior.

## REFERENCES

- [1] Qiu, J., Lang, J.H., Slocum, A.H. A Centrally-Clamped Parallel-Beam Bistable MEMS Mechanism. Proc. IEEE Micro Electro Mechanical Systems (MEMS), 2001.
- [2] Baker, M.S., Howell L.L. On-chip actuation of an in-plane compliant bistable micromechanism. Journal of microelectromechanical systems, 2002.
- [3] Motherway J.E Design Analysis of a Snap-action mechanism. Transactions of ASME, 1976.
- [4] Matoba, H., Ishikawa, T., Kim, C., and Muller, R.S. Bistable Snapping Mechanism. Proceedings of IEEE Micro Electro Mechanical Systems, 1994.
- [5] Vangbo, M., and Bcklund, Y. A Lateral symmetrically bistable buckled beam. Journal of Micromechanical. Microengineering, 1998, v.8.
- [6] Halg, B. On a Nonvolatile Memory Cell Based on Micro-electro-mechanics. Proceedings of IEEE Micro Electro Mechanical Systems, 1990.
- [7] Freudenreich, M., Mescheder, U.M., Somogyi, Design Considerations and realization of a novel micromechanical bistable switch. Proceedings of the 12th international conference on Solid State Sensors, Actuators and Microsystems. Transducers'03, 2003.
- [8] Gomm, T., Howell L.L., and Selfridge R.H., In-plane linear displacement bistable microrelay. Journal Micromechanical engineering, 2002.
- [9] Wagner, B., Quenzer, H.J., Hoershelmann, S., Lisec, T., and Jueress, M., Bistable Microvalve with Pneumatically Coupled Membranes. Proceedings of IEEE Micro Electro Mechanical Systems, 1996.
- [10] Hoffman, M., Kopka, P. and Voges, E., Bistable micromechanical fiber-optic switches on silicon. Proc. 1998 IEEE/ LEOS Summer Topical Meeting, 1998.
- [11] Opdhal, P.G. Modeling and Analysis of Compliant bistable mechanisms using the Pseudo-Rigid-Body Model. M.S. Thesis Brigham Young University, Provo, 1996.
- [12] Jensen, B.D. Identification of macro- and micro-compliant mechanism configurations resulting in bistable behavior. M.S. Thesis Brigham Young University, Provo, 1998.
- [13] Parkinson, M.B., Jensen, B.D., and Roach, G.M. Optimization-Based Design of a Fully-compliant Bistable Micromechanism. Proceedings of the 2000 ASME Design Engineering Technical Conferences, 2000.
- [14] Howell L.L. Compliant mechanisms. Wiley, 2001.