Elastomeric Composites to Reduce the Effects of Trunnion Mode in Inertial Devices

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ABSTRACT

Polymer composites have long been known for their ability to combine the properties of dissimilar materials to create properties ideally tailored for an application. We propose applying these features to MEMS in order to reduce the effects of cross-axis coupling in inertial sensors. By forming aligned composites to create structures rigid in-plane while remaining compliant along the sensitive axis of a MEMS structure, we attempt to reduce the detrimental effects of the trunnion vibrational mode not desirable in unilateral sensors. We model the dynamics of such a system and demonstrate that the use of aligned composites reduces the effects of the trunnion mode by increasing the system's resistance both to angular disturbances and to angular deformation excited by the sense mode due to fabrication imperfections.

Keywords: inertial sensors, polymer MEMS, field-aided composites, cross-axis coupling

1 INTRODUCTION

Proof-mass structures with elastomeric suspensions have been developed by this group for use in high sensitivity inertial sensors with optical detection mechanisms [1]. These proof-masses are designed to be compliant out of the plane of the structure for maximum sensitivity. For such systems it is critical that the proof-mass remains planer in the presence of inertial loading, otherwise sensitivity is degraded. In general, elastomers are isotropic materials and bulk elastomeric suspensions are compliant in both sense and non-sense directions. This may cause the significant presence of higher-order modes during sense mode excitation resulting in non-planer motion. This paper describes a simple extension to the existing fabrication process to create anisotropic elastomer suspensions aligned such that the effects of the out-of-plane torsional or trunnion vibrational mode is reduced and proof-mass planarity in the presence of non-ideal but expected inertial loading and defect cases is enhanced.

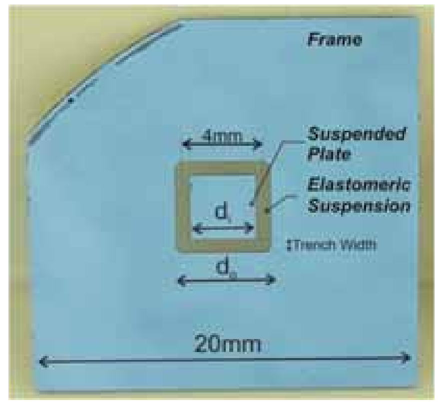

Figure 1: An example of a proof-mass structure with elastomeric suspension.

2 MODELING

2.1 The Shear Suspension

The shear suspension system is a planer structure consisting of a suspended element bonded to a frame by an elastomer annulus (Figure 1). If the out-of-plane thickness of the annulus is greater then its in-plane width, small out-of-plane deflection of the suspended plate is achieved primarily by the shearing of the elastomer [2]. By choosing an elastomer with a low shear modulus (G), high out-of-plane compliance is achieved and, for an inertial sensor based on the deflection of the suspended element, high sensitivity is expected.

In this work, a square annulus is considered. The annulus is decomposed into its rectangular block components. By doing this, the orthogonal stiffness components of suspension system can be considered separately. Exact solutions are known for the stiffness of such bonded blocks in compression/tension, bending and shear [3]. Although exact solutions are known for all simple deformation modes of the circular annulus [4], these results do not extend readily to the case of the anisotropic materials studied here.

2.2 Reduced Order Model

We are principally concerned with the angular, in-plane, and out-of-plane displacement of the proof-mass and the frame. We consider a cross section of the structure to reduce the system to a planer model with six degrees of freedom (DOF) (Figure 2).

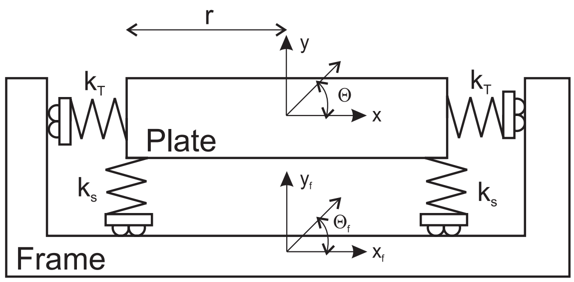

Figure 2: Reduced order model of the suspended plate system.

Each stiffness term represents the stiffness due to one decomposed suspension block element while the subscripts S and T indicate that the mode of deformation is in shear or tension/compression. For example, displacement of the plate in the vertical direction relative to the frame is achieved by the pure shear deformation of the elastomer which can be decomposed into four blocks. Thus, the stiffness is approximated by $4k_S$.

The most significant quantity for sensor systems is the relative displacement between the plate and the frame. By considering the system as such, we reduce it to three DOF with the acceleration of the frame acting as a forcing function. Consideration of this model for small deformations results in the following matrix equation of motion:

$$[M]\begin{pmatrix}\Delta\ddot{x}\\ \Delta\ddot{y}\\ \Delta\ddot{\theta}\end{pmatrix}+[K]\begin{pmatrix}\Delta\ddot{x}\\ \Delta\ddot{y}\\ \Delta\ddot{\theta}\end{pmatrix}=-[M]\begin{pmatrix}\ddot{x_f}\\ \ddot{y_f}\\ \ddot{\theta_f}\end{pmatrix} \qquad (1)$$

where Δ indicates the difference between the plate coordinate and frame coordinate ($\Delta x = x - x_f$, etc.). The mass matrix $[M]$ is

$$[M]=\begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & J\end{bmatrix} \qquad (2)$$

where m is the mass of the plate and J is the mass moment of inertia of the plate about its horizontal axis of symmetry. The stiffness matrix $[K]$ is

$$[K]=\begin{bmatrix} 2k_T+2k_S+\delta_T & 0 & 0\\ 0 & 4k_S+\delta_S & r\delta_S\\ 0 & -r\delta_S & k_t+(2k_S+\delta_S)r^2\end{bmatrix}$$

$$k_t = 2(k_{tT}+k_{tS}) \qquad (3)$$

where δ_T and δ_V are the differences between the right- and left-hand side stiffnesses to account for structural asymmetry due to possible fabrication errors. The k_{tT} and k_{tS} represent the torsional stiffnesses due to tension/compression and shear, respectively, when the plate is deflected in an out-of-plane angular mode. Both terms are lumped together as k_t and this represents the torsional stiffness component not attributable to the deformation of the suspension in simple shear.

2.3 Field-Aided Composites

Oriented short- or chopped-fiber composites form materials with transverse isotropic mechanical properties. This means that in the direction of fiber orientation, the properties of the composite approach that of the fibers alone, while in the transverse direction the properties are similar to that of the matrix. Electrically field-aided composites use high electrical field densities to align particle inclusions immersed in a liquid thermoset polymer into chains along the field lines (Figure 3). When the polymer is cured and set, the inclusion chains act like short-fibers and an elastomer composite is formed [5], [6]. Such composites can be exploited to form structures that are highly compliant in shear and stiff in tension/compression.

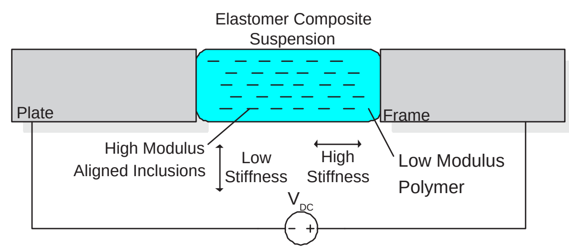

Figure 3: Elastomer inclusion composite aligned in the plane of the suspension system.

Estimations of the composite's aligned Young's modulus (E) and orthogonal shear modulus (G) can be made from simple considerations of the rule of mixtures [7] where the in-plane modulus is given as

$$E = E_iV_i + E_mV_m \qquad (4)$$

where V_i and V_m is the volume fraction of the inclusion and of the elastomer matrix, respectively. For the normal situation where the $E_i \gg E_m$, $E \approx E_iV_i$. Similarly, the G of a shear acting on a plane orthogonal to the direction of inclusion orientation is given as

$$\frac{1}{G}=\frac{V_i}{G_i}+\frac{V_m}{G_m} \qquad (5)$$

For the typical case where $G_i \gg G_m$ and $V_i < V_m$, $G \approx \frac{G_m}{V_m}$.

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Such composites present dramatic elastic property couplings not observed is isotropic elastomers. A simple isotropic elastomer has a well known E/G ratio limit of 3 due to a high Poisson ratio approaching 1/2. The E/G variation is shown in Figure 4 and can be on the order of 10^5 for high modulus inclusions aligned within a very low modulus matrix.

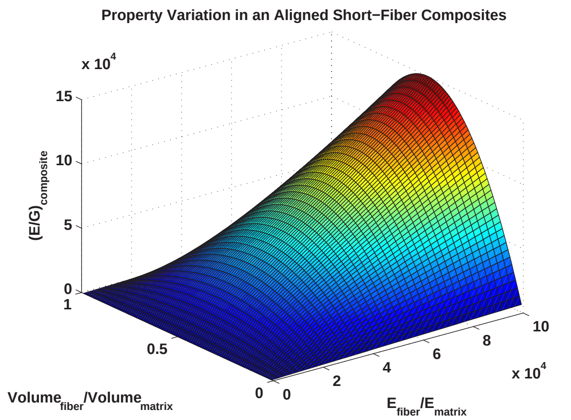

Figure 4: Variation in elastic properties of aligned composites.

For the shear suspension, this high E/G ratio can be exploited if the elastic properties can be suitably aligned with the structure geometry. We see from Eqn. (3) that the in-plane and torsional stiffness terms are a function of the in-plane compressive stiffness and consequently increase with the Young's modulus. The vertical stiffness of the shear suspension is a function only of the shear stiffness and consequently is dependent on the shear modulus. Thus, the use of composites aligned in the plane of the structure reduces in-plane and trunnion mode response while not significantly increasing the sense mode stiffness (Figure 5).

The fabrication procedure currently used to create the elastomer shear suspension has been described in [1]. By mixing in a high modulus inclusion, such as silicon or alumina powder, to the liquid polymer before deposition and curing while applying a 1-10kV/mm field density between the plate and the frame, an aligned composite elastomer is expected to be formed.

3 SIMULATION

Using the MATLAB© numerical package, the linear system of Eqn. (1) is modeled for a suspension composed of a polydimethylsiloxane (PDMS) elastomer matrix with aligned silicon powder inclusions and a bulk silicon frame and plate. The mechanical properties of the commercially available Sylgard®182 from Dow Corning are used (E≈1MPa). The suspended plate structure is

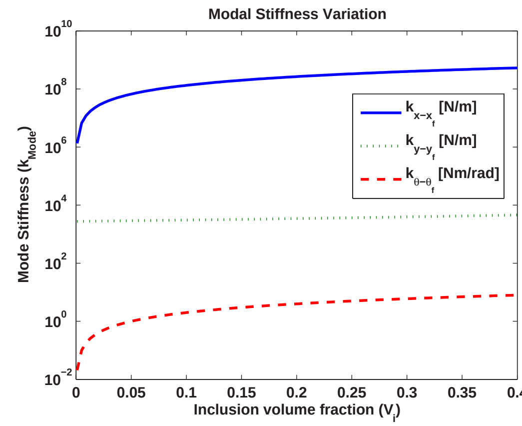

Figure 5: Estimated modal stiffness variation with inclusion volume fraction for a rectangular shear suspension with a 1.0mm plate width and 0.25mm suspension width.

0.5mm thick with a 1.0mm plate width and a 0.25mm wide suspension. The volume fraction of the inclusion, assumed to be perfectly horizontally aligned between the plate and the frame, is varied to explore the effects on the plate dynamics. Structural errors are modeled as a 10 percent asymmetry between the mirrored stiffness components. Damping is introduced by setting the damping ratio of the vertical and horizontal modes to 0.10 via Rayley damping [8].

4 RESULTS

Non-desirable angular deflection of the plate relative to the frame is achieved by two mechanisms in the modeled system. First, if an angular inertial load is placed on the frame, the plate will respond according to it's torsional mass-spring characteristics. In the second mechanism, if the suspension has structural errors that cause the shear spring force to be unbalanced, a moment on the plate will be imposed in the presence of vertical (sense mode) deflection causing cross-axis coupling between the sense and trunnion modes.

The increased resistance of the system to angular inertial load is modeled by studying the response of the system to an unit angular acceleration impulse to the frame. In this case, the trunnion mode response is excited. The addition of aligned inclusions is seen to reduce the amplitude of this response by increasing the stiffness of this mode. Furthermore, this increase in stiffness increased the trunnion mode natural frequency which reduced the settling time and decreasing the time in which the error is experienced (Figure 6).

The addition of aligned inclusions to the shear suspension is also seen to reduce the system's susceptibility to structural errors. Figure 7 shows the trunnion

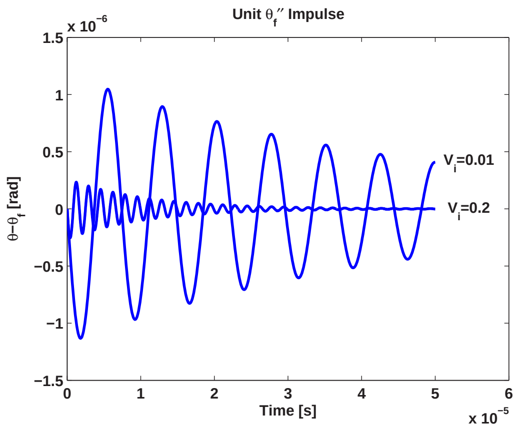

Figure 6: Reduction of trunnion mode response to angular acceleration impulse excitation.

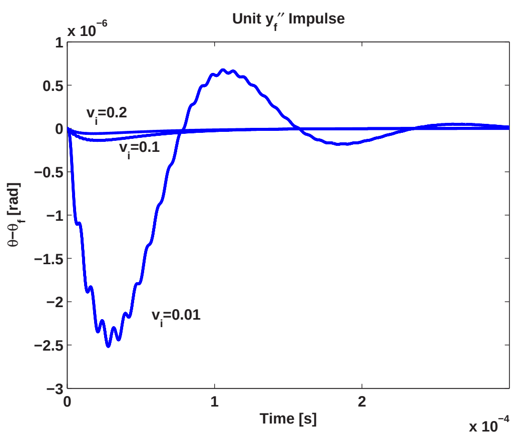

Figure 7: Reduction of trunnion mode response to sense mode (vertical) excitation.

mode response to a vertical acceleration of the frame. The trunnion mode is excited by the asymmetry in the shear suspension and the high frequency response evident in the low inclusion volume fraction case is due to the coupling between the vertical and trunnion modes. By increasing the inclusion volume fraction, and thus the trunnion mode stiffness, the influence of the off-axis terms in Eqn.3 in the dynamic response is reduced and the amplitude of the trunnion mode response is decreased. Furthermore, as in the angular excitation case, the increased natural frequency of the trunnion mode decreases the settling time of the system.

5 DISCUSSION

This study indicates the promise of the use of field-aided elastomer composites in MEMS. Due to the conformal nature of liquid thermoset polymer processing and the unique nature of elastomer mechanical properties, the use of elastomers as mechanical elements in MEMS is long overdue. When coupled to micromachining processes, the use of such elements may provide the next level of sensitivity needed for many sensor applications. One of the difficulties is the low levels of tolerance intrinsic to such bulk processes. The use of the composites discussed here, aligned by the geometry of the structure, have been shown to reduce the dynamic susceptibility to such errors in the specific case of the trunnion mode response. Specifically, the cross-axis sensitivity between the sense and trunnion modes are noted to be reduced. These characteristics have been achieved without a significant reduction in the sense mode compliance of the structure. These concepts further suggest the use of specifically developed structures to tailor the field densities and stiffnesses within the composite elastomer to form elastic properties ideal for the dynamics of a specific MEMS application.

6 ACKNOWLEDGMENTS

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REFERENCES

[1] M. A. Perez and A. M. Shkel. Conceptual design and preliminary characterization of serial array system of high-resolution mems accelerometers with embedded optical detection. *Smart Structures and Systems*, 1(1):63–82, January 2005.

[2] P. M. Sheridan, F. O. James, and T. S. Muller. Design of componets. In A. N. Gent, editor, *Engineering with Rubber*, chapter 8, pages 224–253. Hanser, 2nd edition, 2001.

[3] A. N. Gent and E. A. Meinecke. Compression, bending, and shear of bonded rubber blocks. *Polymer Engineering and Science*, 10(1):48–53, 1970.

[4] D. F. Sims. Asymmetric loading of bonded rubber cylinders. *Polymer Engineering and Science*, 15(9):652–659, September 1975.

[5] G. H. Kim, D. K. Moeller, and Y. M. Shkel. Functionally graded polymeric composites having microtailored structure fromed by electric field. In *Proceedings of IMECE'03 (IMECE2003-43518)*. ASME, November 2003.

[6] G. H. Kim and Y. M. Shkel. Polymeric composites tailored by electric field. *J. Mater. Res.*, 19(4):1164–1174, April 2004.

[7] B. Z. Jang. *Advanced Polymer Composites: Principles and Applications*. ASM Internations, 1st edition, August 1994.

[8] A. K. Chopra. *Dynamics of Structures*. Prentice Hall, 1st edition, 1995.