Snap-Action Bistable Micromechanism Actuated By Nonlinear Resonance

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***Abstract*— Snap-action bistable mechanisms have a practical use in applications when very few but well defined states of a micro-mechanism are required. Conventionally, the switching between bistable states is done statically, utilizing either electrostatic or thermal actuation. This paper explores a paradigm utilizing structural resonance phenomena to switch dynamically between states. We reports a detailed mathematical model governing the non-linear response of the buckled beams in their bistable equilibrium; analytical and FEA results describing non-linear dynamics of the mechanism near its equilibrium state and transient dynamics of switching between bistable states; and design, fabrication, and initial characterization results of a micromachined double-beam test structure.**

I. Introduction

Snap-action bistable mechanisms have a practical use in applications when very few but well defined states of a micro-mechanism are required. Some obvious applications include micro-switches, addressable MEMS-based pixel arrays, and tunable optical MEMS filters. An advantage of snap-action mechanisms is that no power is needed to keep the mechanism in either of its bistable states. Conventionally, the switching between bistable states is static, utilizing either static or thermal actuation. For a typical size micro-device, 100s volts are needed to switch statically from state to state. This paper explores a paradigm utilizing structural resonance phenomena to switch dynamically between states. If a linear structure is driven into resonance, the structure can achieve a relatively large amplitude of oscillation using a relatively small amplitude actuation force. In context of a bistable mechanism, under certain conditions, the structure driven into resonance-like vibration may achieve a large enough deviation from its equilibrium, sufficient to switch between states. We have explored the dynamic switching phenomena analytically and demonstrated experimentally the predicted dynamic behavior of structures near the bi-stable equilibrium state. This paper reports on (i) a detailed mathematical model governing the non-linear response of the buckled beams in their bistable equilibrium, (ii) analytical and FEA results describing non-linear dynamics of the mechanism near its equilibrium state and transient dynamics of switching between bistable states, and (iii) design, fabrication, and characterization results of a double-beam test structure in the neighborhood of the equilibrium state.

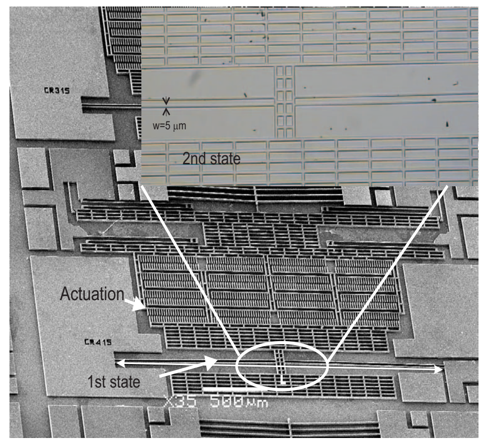

Fig. 1. SEM pictures of the fabricated devices using SOI -based process

The study of dynamic behavior of bistable micro-mechanism is a new class of problems in the MEMS field. Though in the civil engineering community researchers have been studying the dynamics of nonlinear structures, such as shallow sinusoidal arches, for many years. Of the special interest were geometry of civil structures that could be subjected to dynamic loads, such as wind. For example, Tseng and Dugundji (1971) [1], noticed the complexity of the buckled beam's behavior when put under forced oscillations. They reported the presence of snap-through phenomenon. In 1980, Yamaki and Mori,[2], noticed the presence of internal resonance and combination of resonance in buckled beams made of commercial duralumin sheet of 0.5 mm thickness.

This type of bistable structures are very useful design elements, but still rarely found in MEMS. In [7], we have already analyzed the use of dynamic switching and its power consumption advantages for a special class of bistable structures. In this paper, we expand the class of structures and analyze the same phenomena but for clamped-clamped double-coupled in plane buckled beam. This type of design offers the same actuation possibility as in [7], but is more symmetric and can be actuated at lower voltages for the same device footprint. The devices analyzed in this paper have been fabricated using an SOI-based process, figure 1.

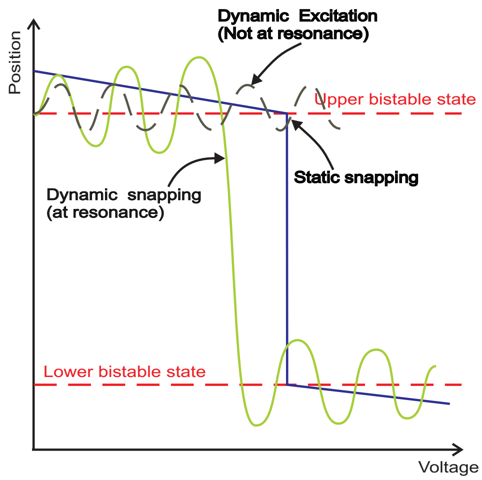

Fig. 2. Static and dynamic switching strategies

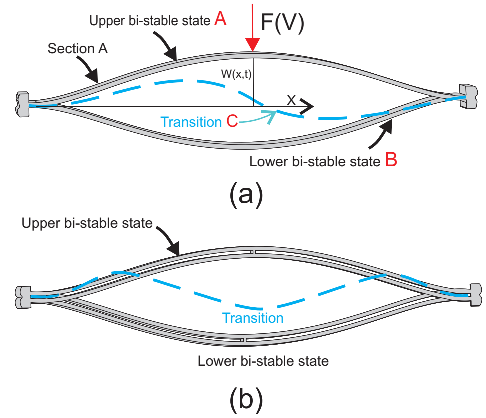

Fig. 3. Illustration of the clamped-clamped curved beam

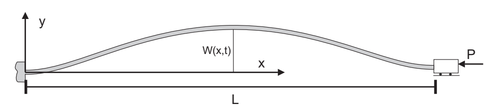

Fig. 4. Schematics of the deformed shape of a single beam after applying a buckling load

On a micro-scale this type of structural elements is well-suitable for relays and switches, addressable MEMS-based pixel arrays, and tunable optical MEMS filters. In [4], Jin Qiu proposed the use of bi-stable mechanisms in the microworld as a microswitch and gave a detailed analytical comprehension to the properties and behavior of this type of structures, though only considering the static behavior.

In this paper, we analyze the design approach of utilizing an electrostatic actuation of bi-stable system which allows dynamic switching between the discrete bi-stable states. The advantage of using dynamic forces is to reduce power consumption by driving the device to instability (an equivalent to resonance type behavior for linear systems at its natural frequency), so that the amplitude of motion is maximized when injected in the system under well specified conditions. In figure 2, we model the response illustrating that the switching is achieved at lower voltage than when actuated statically. In the dynamic case, we are specifically interested when the switching occurs when the device is actuated at the resonance frequency.

The dynamics of this type of structures have been analyzed carefully by Nayfeh et al., [3]. They obtained an exact solution of the buckling mode shapes and later, in [5], Lacarbonara compared the Galerkin discretization and the direct application of a reduction method to the original governing equations of a buckled beam with experimental results. Samir A. Emam [6] investigated theoretically and experimentally the nonlinear responses of a clamped-clamped buckled beam to a variety of external harmonic excitations and internal resonances. This paper expands the dynamic actuation results to micromachined structures.

II. Design Analysis

The system presented in figure 3 (a) does not use latches, or hinges, or residual stresses to achieve its bistability. This mechanism can be built already buckled without prestress with one mask DRIE process and used as a MEMS relay directly [4]. Although the system may buckle and snap with and external force F(V) applied at the center point, it will not stay in the snapped shape when the force is released, due to a twisting buckle first mode. However, this mode in two single curved beams is mutually canceled yielding to a bistable double-beam without prestress, see figure 3 (b). The restoring force of a bistable double-beam is exactly twice larger than for a single beam, though the mechanical behavior is the same with the odd modes restricted.

In order to obtain an analytical model we studied the single beam, such that the one in figure 4, with area A, moment of inertia I, modulus of elasticity E, and beam length L.

The beam is modeled according to the Euler-Bernoulli theory, assuming that the cross-section is considered uniform and its material is homogenous. This gives the differential equation of motion governing the transverse vibrations of the beam when the system is subjected to an electrostatic excitation due to the electrostatic comb drive actuation with the frequency Ω. The transverse equation of motion becomes

$$m\frac{\partial^2\overline{\omega}}{\partial\overline{t}^2}+EI\frac{\partial^4\overline{\omega}}{\partial\overline{x}^4}+P\frac{\partial^2\overline{\omega}}{\partial\overline{x}^2}+c\frac{\partial\overline{\omega}}{\partial\overline{t}}$$

$$-\frac{EA}{2L}\frac{\partial^2\overline{\omega}}{\partial\overline{x}^2}\int_0^L(\frac{\partial\overline{\omega}}{\partial\overline{x}})^2dx=\overline{F}\cos^2\overline{\Omega}\overline{t} \qquad (1)$$

subject to the boundary conditions $\overline{w}=0$ and $\frac{\partial\overline{\omega}}{\partial\overline{x}}=0$ at $\overline{x}=0$ and $\overline{x}=L$. Here the overline means dimensional quantities and F is the electrostatic force due to the comb drives ($\overline{F}=\frac{1}{2}N\varepsilon_0\frac{t}{g}V^2$), where V is the potential difference (voltage) applied between combs, N the number of gaps, g the gap between combs, t the thickness of the structure, and $\varepsilon_0=8.854x10^{-12}F/m$. In this expression, c is the damping coefficient which is mainly due to the viscous effects of the air between the mass and the substrate and between the comb-drive capacitor fingers.

To simplify the equation, the following non-dimensional variables are used:

$x = \frac{\overline{x}}{L}$, $\omega = \frac{\overline{\omega}}{L}$, $t = \overline{t}\sqrt{\frac{EI}{mL^4}}$, and $\Omega = \overline{\Omega}\sqrt{\frac{mL^4}{EI}}$, where $r = \sqrt{\frac{I}{A}}$ is the radius of gyration of the cross-section. As a result, the equation can be simplified as follows

$$\ddot{\omega} + \omega^{iv} + P\omega'' + c\dot{\omega} - \frac{1}{2}\omega'' \int_0^1 \omega'^2 dx = F cos^2(\Omega t), \quad (2)$$

where $w = 0$ and $\omega' = 0$ at $x = 0$ and $x = 1$. In this equation, the overdot indicates the derivative with respect to time t, the prime indicates the derivative with respect to the spatial coordinate x, and $P = \frac{\overline{P}L^2}{EI}$, $c = \frac{\overline{c}L^2}{\sqrt{mEI}}$ and $F = \frac{\overline{F}L^4}{rEI}$ are non-dimensional quantities.

From [6], Galerkin method is suitable to model microsized devices. In this study, multi-mode Galerkin discretization is utilized and linear vibration mode shapes of the buckled beam are used as a trial function. Since MEMS devices usually have the two first modes very close and due to the internal resonances that a curved beam presents, it is reasonable keeping the two modes in the discretization. The behavior around the buckled state is described by a summation of the product of the linear vibration mode shapes of the buckled beam $\phi_n(x)$, by the generalized coordinates $q_n(t)$:

$$\omega_b(x,t) = \phi_1(x)q_1 + \phi_2(x)q_2, \quad (3)$$

where for an initially buckled beam $\phi_1 = b/2(1 - cos2\pi x)$ and $\phi_2 = b(2x - \frac{2}{2.86\pi}sin(2.86\pi x) + cos(2.86\pi x) - 1)$. If b is the initial buckling height, then the vertical motion of the beam is described by

$$\omega(x,t) = b/2(1 - cos2\pi x) + \phi_1(x)q_1 + \phi_2(x)q_2 \quad (4)$$

In this expression q_1 and q_2 are derived from solving the set of equations obtained from substituting 4 into 2 and applying the Galerkin method:

$$\ddot{q_m} + \omega_m^2 q_m = -c\dot{q_m} + b\sum_{i,j}^{N} A_{mij}q_iq_j + \sum_{i,j,k}^{N} B_{mijk}q_iq_jq_j + f_m cos^2\Omega t \quad (5)$$

$m = 1, 2$

where all the coefficients are referenced in table I, and f_1 and f_2 are calculated as:

$$f_1 = \int_0^1 F\phi_1 dx = \frac{\varepsilon NtbV^2}{4g} \quad (6)$$

$$f_2 = \int_0^1 F\phi_2 dx = \frac{4.34 \times 10^{-5}\varepsilon NtbV^2}{g} \quad (7)$$

These equations model the dynamic behavior around the buckled position of a single buckled beam. The dynamic restoring force response of the double buckled beam is exactly twice larger than those of a single buckled beam.

TABLE I

COEFFICIENTS.

Coeff.	Value	Coeff.	Value	Coeff.	Value
A_{111}	$-36.528b^3$	B_{1111}	$-12.17b^4$	B_{2111}	$0.0083b^4$
A_{121}	$0.025b^3$	B_{1211}	$0.0083b^4$	B_{2211}	$-99.63b^4$
A_{112}	$0.025b^3$	B_{1121}	$0.0083b^4$	B_{2121}	$-5.6e-6b^4$
A_{122}	$-99.63b^3$	B_{1221}	$-5.6e-6b^4$	B_{2221}	$0.0677b^4$
A_{211}	$0.025b^3$	B_{1112}	$0.0083b^4$	B_{2112}	$-5.6e-6b^4$
A_{221}	$-199.26b^3$	B_{1212}	$-5.6e-6b^4$	B_{2212}	$0.0677b^4$
A_{212}	$-1.7e-5b^3$	B_{1122}	$-99.63b^4$	B_{2122}	$0.0677b^4$
A_{222}	$0.2031b^3$	B_{1222}	$0.0677b^4$	B_{2222}	$-815.2b^4$

TABLE II

DIMENSIONS OF THE BISTABLE MICROMECHANISM.

Dimensions	Description	Value
L	Length	$3500\mu m$
t	thickness	$70\mu m$
w	In-plane width	$5\ \mu m$
b	Initial height	$13\ \mu m$

III. RESULTS

A. Statics

The devices studied in this paper were fabricated using an SOI-based micromaching process. The parameters chosen to implement the device are in Table II.

The static behavior has been analyzed using ANSYS FEA analysis. The geometry of the device was parameterized and meshed using a planar 2D BEAM3 elements. A very careful mesh density was selected, as the design is geometrically nonlinear and the system has two stable states in the range of displacements analyzed, figure 5. In order to obtain accurate results, the arc-length method is used to obtain the force response. This method is suitable for nonlinear static equilibrium solutions of unstable problems.

The modeling results were used as a guide for synthesizing a design suitable for low actuation force operation. The critical parameters in the design are the geometry of the mechanisms, placement and geometry of actuation electrodes, and apex between bi-stable states of the structure. In our implementation of the mechanism, figure 5, we see that the maximum force needed to statically switch from the initial state to the final state is about 15 μN, or 20 V applied statically, and the motion range is 26 microns.

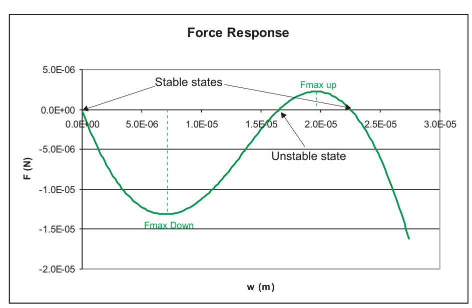

Fig. 5. Force response of the clamped-clamped curved beam when a static force is applied at the central point

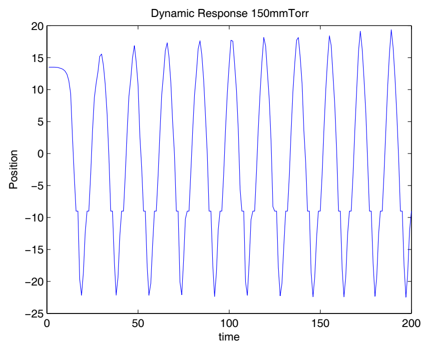

Fig. 6. Chaotic behavior of the buckled beam in vacuum 150 mmTor

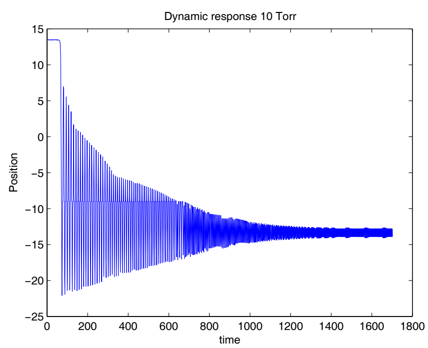

Fig. 7. Snap-through behavior of the buckled beam at 10 mmTor

B. Dynamics

The same micromechanism was now driven by an electrostatic force with an alternating harmonic voltage applied at the resonance frequencies. The force is applied through an electrostatic comb drive structure. When the frequency of the external force was equal to the resonance frequency of the device, the system accumulated enough energy to switch between states at a voltage lower than the static snapping voltage. In figures 6 and 7 the system is driven by a sinusoidal voltage with 4 V peak-to-peak amplitude. Depending on the damping, the behavior of the system can become chaotic, see figure 6, where the device oscillates between the two stable states without becoming stable due to low energy losses. At higher damping values, see figure 7, the system presents snap-through behavior and the actuation pick-to-pick voltage is much lower (4V) than the voltage needed to switch between states statically (20V).

C. Experiment

We designed, fabricated, and characterized bistable test structures. The devices are fabricated using an SOI-based process developed by the group. In figure 1, an SEM of the device is shown. In the fabricated designs, the length of the beams varies from 2000 μm to 3500 μm, the beams are 70 μm thick and 5 μm wide, and the initial apex of the buckled beam changes from 10 μm to 15 μm. The bistable test structures are actuated electrostatically using comb drive microactuators to avoid snapping. A technique for enhancement of electrode spacing was used in the designs.

The structures were characterized in a vacuum chamber under 200mTorr pressure. To demonstrate the dynamically triggered switching between bi-stable states, the structures were driven by a sinusoidal voltage $V_{AC} = 3.53V$.

The response of devices was detected both electrostatically and optically. The experimental measurements demonstrated the non-linear behavior of the resonators with the central frequency near 6.8kHz, closely matching analytical and FEA results.

IV. Conclusion

In this paper we explored the use of nonlinear structural resonance phenomena to switch between states of a bistable structure. It was demonstrated that this type of switching results in at least 40 % savings in power consumption as compared to the static switching. In order to test this switching strategy, an initially built buckled beam without pre-stress was implemented. Multi-mode Galerkin method was used to describe the analytical response of the device. The feasibility of driving the micromechanism into resonance and achieving dynamical switching has been modeled. It was illustrated, that dynamic switching strategy reduces the power consumption for a micromechanism of the dimensions specified. The conclusion is supported by analytical models confirming that when the system is driven dynamically (with a sinusoidal voltage using a comb drive actuator only), the pick-to-pick voltage of only $V_{AC} = 4V$ is needed to switch between the states, whereas when driven statically (thermal or electrostatic actuation) higher voltages in the order of $20V$ is required. The parameters of analytical models and bi-stable behavior have been verified using FEA modeling. Several tests structures of the mechanism were fabricated using an SOI-based process, and the behavior near the equilibrium was tested and agree with the analytically predicted behavior.

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