On Electrostatic Actuation Beyond Snapping Condition

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***Abstract*— Electrostatic parallel-plate actuators are a common way of actuating MEMS devices, both statically and dynamically. In the static case, the actuation range is limited to 1/3 of the initial actuation gap, known as the static pull-in condition. Under dynamic actuation conditions, however, the travel range can be much extended. This paper extends the analysis of pull-in instability to the dynamic case and derives the analytical *AC Dynamic Pull-in Condition*. This condition predicts snapping or pull-in of the structure for a given domain of DC and AC actuation voltages versus Quality factor. Analytical and experimental results are presented to validate the dynamic pull-in condition.**

I. INTRODUCTION

MEMS sensors based on electrostatic parallel-plate actuation principle are seemingly simple to fabricate and operate while providing relatively large forces. However, it is well-known the instability resulting from the non-linearity of the parallel-plate electrostatic force that happens at 2/3 of the initial gap, known as the *Static Pull-in Condition*.

Few comprehensive studies analyze the effect of dynamics on the stability of the parallel-plate electrostatic actuation, even though the effects have been identified as important for such devices as electrostatically actuated microrelays [1] or rate integrating gyroscopes [2]. This paper attempts to fulfill this gap in the literature.

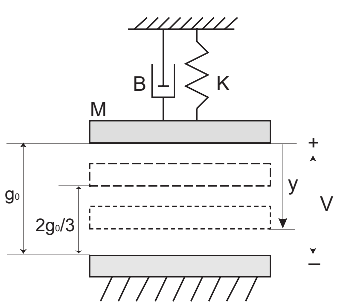

Fig. 1. Schematic of the electromechanical system with parallel plate actuation. The static pull-in occurs when the distance between plates is $2/3$ of the initial gap. In the dynamic case, we are interested in expanding the maximum achievable amplitude of oscillation, **y**, beyond static snapping condition.

II. PULL-IN ANALYSIS

Consider a lumped mass-spring-damper model (Fig. 1), with the usual assumption of voltage-controlled actuation. This is a common representation of a wide variety of MEMS devices. We observed that with dynamically moving structures actuated electrostatically, the well-known static pull-in or snapping condition is changed. The reason is that the kinetic energy and the dissipation of energy are now playing an important role in defining the snapping condition.

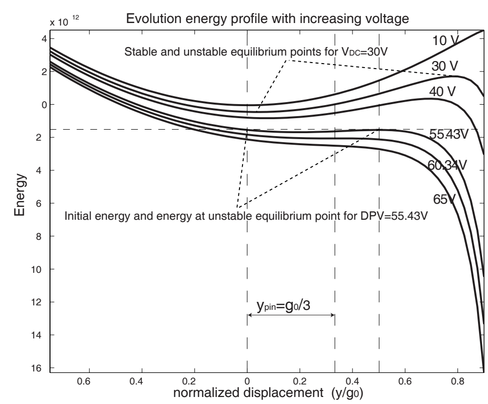

Fig. 2. The Potential Energy level of the analyzed system depends on the position relative to the gap. In this example, energy of the system versus normalized displacement for different applied voltages are displayed, including the *Static Pull-in Voltage* (60.34 V) and the *Dynamic Pull-in Voltage* (55.43 V).

To explain the phenomena, consider the energy of the electro-mechanical system

$$E = E_k + E_p + U = \frac{1}{2}M\ \dot{y}^2 + \frac{1}{2}K\ y^2 - \frac{1}{2}\frac{\varepsilon_0\ A}{(g_0 - y)}V^2 \quad (1)$$

which is composed of the kinetic energy (E_k), the potential energy stored in the spring (E_p), and the potential energy stored in the parallel plate capacitor (U). The study of the evolution of the energy can be used to determine the equilibrium positions of the system, as well as the regions of instability.

In the static equilibrium, $\ddot{y} = \dot{y} = 0$, the energy of the system (1) consists only of the potential energy terms. As a result, the distribution of the system energy along the gap between the electrodes is constant and unique for each voltage applied (Fig. 2).

Analyzing the energy profile at its equilibrium points, the limiting condition for existence of a stable equilibrium is the presence of an inflection point in (1). This condition provides the analytical value for the maximum static stable displacement from the initial equilibrium and the voltage needed to reach this position

$$y_{pin} = \frac{g_0}{3} \ ; \ SPV = \sqrt{\frac{8}{27} \frac{K\, g_0^3}{\varepsilon_0\, A}} \tag{2}$$

This voltage is called the *Static Pull-in Voltage* (SPV).

The energy analysis can be expanded to account for the transient dynamics of the system when an actuation voltage is applied.

The time derivative of the system energy defined in (1),$\frac{dE}{dt} = -B\, \dot{y}^2$, indicates that if energy is not continuously pumped into the system, the energy decreases with time from its initial energy value until it reaches an equilibrium state, $\frac{dE}{dt} = 0$. According to the model, the only factor that defines the pattern of the energy decay is the damping, B, of the system.

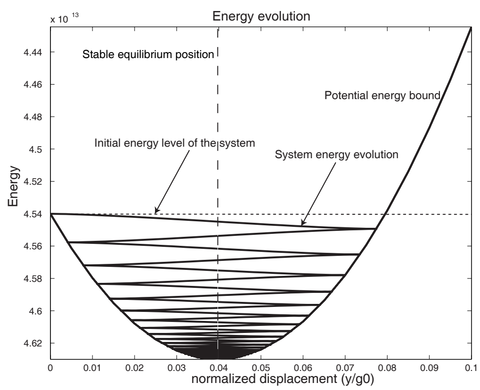

Fig. 3. Evolution of system's energy when a 30 V step-function is applied. The Quality Factor in this example is 30. The initial energy corresponds to the potential energy (mechanical and electrostatic). When the motion begins, the potential energy is converted to kinetic energy and dissipated due to damping forces. The system's energy descends until it reaches the stable equilibrium position.

As can be observed in Fig. 3, the maximum amplitude of displacement of the moving plate is limited by the potential energy bound. If the voltage is increased, at some point the initial energy of the system and the energy at the unstable peak have the same magnitude (Fig. 2). Assuming that the system has no damping forces, the total energy of the system remains constant, what implies that applying a higher voltage, the system will move until it overshoots the unstable equilibrium, and the electrodes will collide. This voltage limit is called *Dynamic Pull-in Voltage* (DPV). Any voltage lower than DPV magnitude cannot produce snapping. Analytically, the voltage limit has the unstable equilibrium at the center of the gap and is described by the following expression

$$y_{uns} = \frac{g_0}{2} \ ; \ DPV = \sqrt{\frac{1}{4} \frac{K\, g_0^3}{\varepsilon_0\, A}} \tag{3}$$

Analysis of the pull-in voltage against quality factor shows that in over-damped systems the value needed to produce snapping corresponds to the *Static Pull-in Voltage* and that this value decreases with the increase of the Quality Factor (Q) until it settles at the *Dynamic Pull-in Voltage*. Similar results were presented in [3], where the numerical simulation was confirmed with experimental results.

III. AC DYNAMIC PULL-IN CONDITION

In those cases where the system is dynamically actuated, as in resonators, accelerometers or gyroscopes, the stability analysis becomes more complex. Under forced oscillation, the voltage varies with time, $V(t) = V_{DC} + V_{AC}(t)$, meaning that the energy equilibrium points given by $\frac{dE}{dy}$ are changing continuously

$$\frac{dE}{dy}(t) = K \cdot y - \frac{1}{2} \frac{\varepsilon_0\, A}{(g_0 - y)^2} V(t)^2 = 0 \tag{4}$$

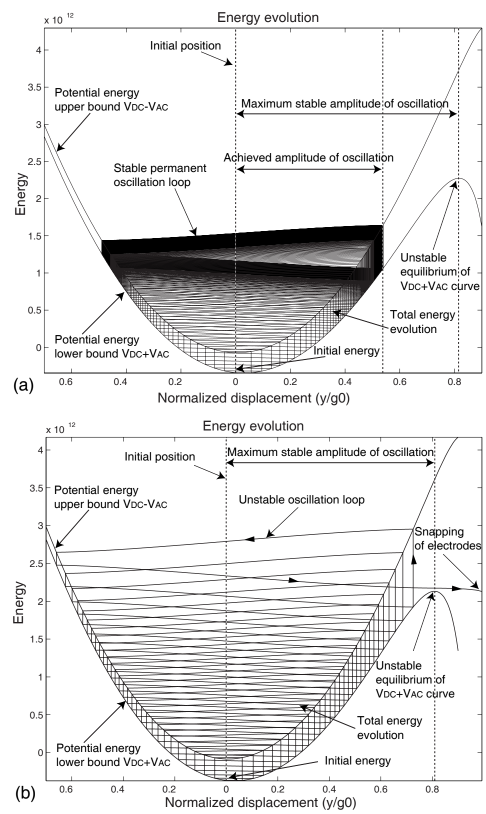

Fig. 4. The potential energy curves bound the system oscillation. In (a) a stable oscillation is obtained with 19 V_{DC} bias voltage and a 7 V_{AC} amplitude while in (b) the oscillation is unstable with 20 V_{DC} and 7 V_{AC}. Beginning from the static initial position, the amplitude of oscillation increases until it reaches the unstable equilibrium point at $V_{DC} + V_{AC}$, resulting in snapping.

As in the dynamic case, the potential energy curves bound the evolution of the total energy of the system. Consequently, their analysis allow to determine the maximum amplitude of oscillation that can be achieved without reaching the pull-in zone (Fig. 4). Equation (4) provides the condition for the extreme points of the energy function. Solving the equation for $V(t) = V_D + V_{AC}$, and discriminating maximum and minimum points using the second derivative, we can define y_{uns} as the unstable equilibrium (maximum) of the $V_D + V_{AC}$ potential energy curve. Amplitudes smaller than y_{uns} are stable, while larger amplitudes lead to pull-in [4].

Consequently, once the maximum stable amplitude is determined, the next step is to predict if the chosen driving voltage, $V(t) = V_{DC} + V_{AC}(t)$, will drive the system to pull-in or a stable oscillation loop will exist.

If the alternating voltage V_{AC} is considered to be a square-function (without lost of generality the case can be extended to other driving functions), at each half period the system behaves like in the dynamic case when a constant load is applied. When the voltage changes, the energy of the system jumps to the other energy region (Fig. 4).

During the oscillating loop, the energy decay is controlled by the damping constant (B). Assuming that the oscillation is sinusoidal, $y(t) = y_A sin(\omega t)$, the energy lost due to damping forces at each half period would be

$$E_{lost} = -By_A^2 \omega \frac{\pi}{2} \tag{5}$$

where y_A is the amplitude of oscillation and ω is the resonant frequency of oscillation of the system.

Consequently, the stability of oscillation will depend on the energy balance between the gained energy due to $V(t) = V_{DC} + V_{AC}(t)$ actuation and the energy losses due to damping [5].

In an energy oscillation loop, four energy terms are considered: E_1, E_2, E_3, E_4 (Fig. 5). The initially gained energy (E_1), when moving from $V_{DC} + V_{AC}$ curve to $V_{DC} - V_{AC}$ curve is

$$\begin{aligned} E_1 &= \left[\frac{1}{2}K\ (y_{st} + y_A)^2 - \frac{1}{2}\frac{\varepsilon_0\, A(V_{DC} - V_{AC})^2}{(g_0 - (y_{st} + y_A))}\right] \\ &- \left[\frac{1}{2}K\ (y_{st} + y_A)^2 - \frac{1}{2}\frac{\varepsilon_0\, A(V_{DC} + V_{AC})^2}{(g_0 - (y_{st} + y_A))}\right] \\ &= \frac{2\,\varepsilon_0\, A\, V_{DC}\, V_{AC}}{(g_0 - y_{st} - y_A)} \end{aligned} \tag{6}$$

where y_{st} is the position displacement of the electrode due to the V_{DC} bias. In this expression, $y_{st} + y_A$ represents the effective position in the gap.

The energy losses due to damping during the $V_{DC} - V_{AC}$ half-period (E_2) and the $V_{DC} + V_{AC}$ half-period (E_4)

$$E_2 = -By_A^2 \omega \frac{\pi}{2}\,;\ \ E_4 = -By_A^2 \omega \frac{\pi}{2} \tag{7}$$

And the energy reduction when moving from $V_{DC} - V_{AC}$ curve to $V_{DC} + V_{AC}$ curve, obtained in a similar way as in (6)

$$E_3 = -\frac{2\,\varepsilon_0\, A\, V_{DC}\, V_{AC}}{(g_0 - y_{st} + y_A)} \tag{8}$$

If the system could be actuated at a stable resonant frequency, there must exist an amplitude of oscillation where the

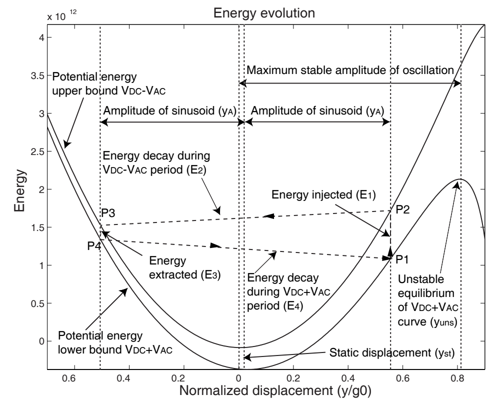

Fig. 5. *AC Dynamic Pull-in calculation*. In the plot, the actuation voltage is stable and generates a stable loop. The energy injected by the actuation is balanced with the damping losses creating a stable oscillation loop.

energy balance of the loop must be zero.

$$\begin{aligned} E_1 + E_2 + E_3 + E_4 &= \\ \frac{4\,\varepsilon_0\, A\, V_{DC}\, V_{AC}\, y_A}{(g_0 - y_{st})^2 - y_A^2} - By_A^2 \omega\pi &= 0 \end{aligned} \tag{9}$$

Rearranging terms in (9), the amplitude of oscillation, y_A, of the stable loop can be obtained from the following equation

$$y_A^3 - (g_0 - y_{st})^2 y_A + \frac{4\,\varepsilon_0\, A\, V_{DC}\, V_{AC}}{B\omega\pi} = 0 \tag{10}$$

The equation can be solved analytically. However, to predict the existence of stable oscillation, we only need to know the type of solutions of equation (10). This analysis can be done through the 3rd order polynomial discriminant, D, of the equation

$$D = -\frac{1}{27}(g_0 - y_{st})^6 + \frac{4\,\varepsilon_0^2\, A^2\, V_{DC}^2\, V_{AC}^2}{B^2 \omega^2 \pi^2} \tag{11}$$

In a cubic polynomial, $D = 0$ identifies the transition between all-real solutions and the existence of complex solutions. Applied to the parallel-plate system, this equation leads to the *AC Dynamic Pull-in Condition (ACPC)*

$$ACPC = V_{DC}\, V_{AC} = \frac{B\omega\pi(g_0 - y_{st})^3}{6\,\sqrt{3}\varepsilon_0\, A} \tag{12}$$

that provides the maximum value of the product $V_{DC}\, V_{AC}$ producing stable oscillation.

The *AC Dynamic Pull-in Condition* defines a constructive domain of V_{DC} and V_{AC} actuation voltages versus Quality factor preserving stability of the parallel-plate actuation.

IV. EXPERIMENTAL RESULTS

As an example, to validate the energy analysis and the *AC Dynamic Pull-in Condition* we examined the structure in Fig. 6. The structural parameters are as follow: Stiffness is $K = 3.1\ N/m$, mass is $M = 3.76e^{-12}\ Kg$, initial gap is $g_0 = 2\ \mu m$, and area of parallel-plate actuator is $A = 228\ \mu m^2$. For this structure, the static instability (2) occurs at $60.34\ V$, when the gap becomes approximately $1.3\ \mu m$. No equilibrium points exist at smaller gap for voltages higher than *Static Pull-in Voltage*, as can be observed in Fig. 2.

When the actuation voltage is applied as a non-smooth function of time, as it would be if the voltage were applied as a step function, the *Dynamic Pull-in Voltage* is 55.34 V, (3). As can be observed, the voltage is approximately 8 % lower. In this case, the snapping for voltages higher than this value will depend on the damping of the system, which is directly proportional to the air pressure of packaged micro-devices. For low pressure or vacuum conditions, voltages higher than DPV would imply snapping.

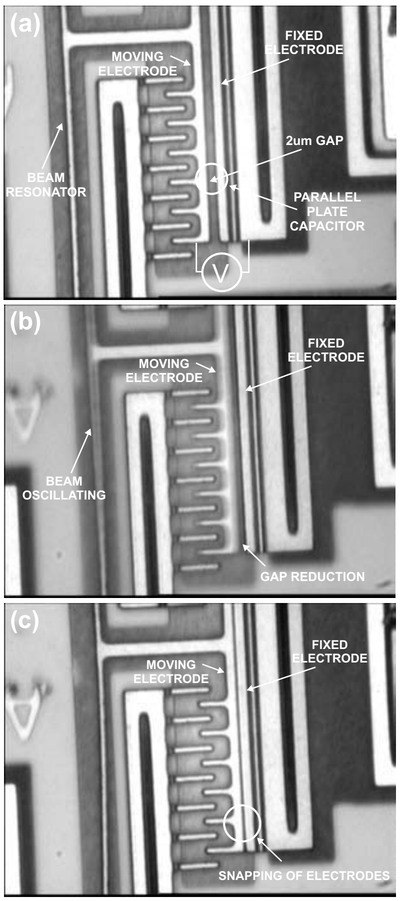

Fig. 6. The set of pictures presents an evolution of the amplitude of oscillation of the resonator due to changes of AC-DC driving voltages; The beam is oscillated using the parallel plates electrodes, while the lateral combs are disabled; The *Static Pull-in Voltage* is 60.34 V. (a) Beam at rest; (b) Beam oscillating with 15 V_{DC} and 7 V_{AC}; (c) Beam snapped after a combination 20 V_{DC} and 7 V_{AC} drive voltages are applied .

It should be noted that when the structure is dynamically actuated to its resonant frequency, the instability (or snapping) occurs at much smaller voltages. In the example presented, snapping occurred at 20 Volts DC-bias and 7 Volts pick-to-pick AC-amplitude, having achieved larger actuation amplitude, or equivalently smaller gaps, approximately 0.2 μm in our case (with 2 μm of the nominal gap). As can be observed, significantly larger amplitude of actuation can be achieved when dynamic actuation is used. The 'overshoot' effect of the static equilibrium is explained by the gained kinetic energy of the system which allows it to return to the stable region of actuation.

The experimental values obtained with the system were used to validate the energy analysis predictions. In Fig. 7, the *AC Dynamic Pull-in Condition* has been used to produce the combination of maximal allowed V_{DC} and V_{AC} voltages for Quality Factor ranging from 20 to 30. As can be observed, the analytical predictions are compared with experimental results for the system in Fig. 6. Experimental data is consistent with the analytically derived regions of instability.

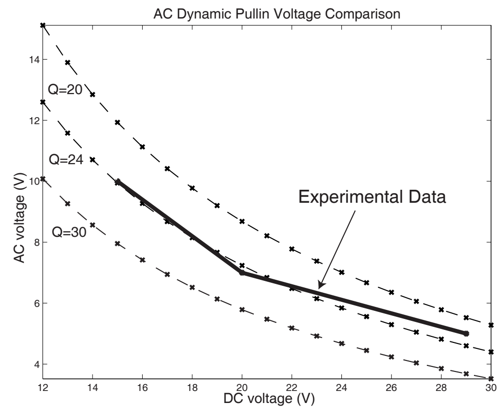

Fig. 7. Maximum combinations of V_{DC} and V_{AC} voltages for the different values of the Quality Factor (Q). Values estimated with the *AC Dynamic Pull-in Algorithm* are presented with the experimental data. The trend of experimental data agrees with the predictions.

V. Conclusions

Operation of electrostatically actuated MEMS with amplitudes much higher than 1/3 of the initial actuation gap can be achieved with the appropriate selection of actuation voltages. The system kinetic energy gained during dynamic actuation allows the system to travel beyond the static equilibrium, reaching large amplitudes of oscillation without snapping.

Energy analysis has been used to derive the *AC Dynamic Pull-in Condition* which provides the combination of maximum V_{DC} and V_{AC} voltages that can be used to actuate the system without producing snapping at resonance frequency.

Acknowledgment

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