Effect of Fabrication Imperfections on Energy Loss through Mechanical Mode Coupling in MEMS

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***Abstract*—We identified the mechanical mode coupling as a major energy loss mechanism limiting the quality factor of MEMS resonators, such as gyroscopes. An analytical model was derived to quantify the effect of fabrication imperfections on the phenomenon of intermode energy exchange. Using both finite element analysis and analytical modeling, the mechanical coupling between in-plane and out-of-plane modes of resonators with fabrication imperfections was studied and the decrement in the quality factor due to mode coupling was predicted. The analysis was used to explain why some low frequency designs are exhibiting a quality factor close to their fundamental TED limit, while others are showing the quality factor with orders of magnitude lower than their TED limit. Our analytical predictions are supported experimentally.**

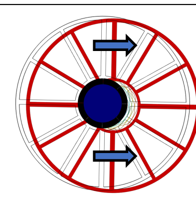
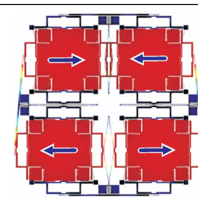
I. Introduction

Various energy loss mechanisms have been reported to influence and limit the quality factor of MEMS resonators, [1,2]. Viscous air damping, thermoelastic damping, and anchor loss are examples that play an important role in setting the quality factor (Q-factor) limit in MEMS resonators. The total Q-factor of a certain mode can be expressed as:

$$\frac{1}{Q_{total}} = \frac{1}{Q_{viscous}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{anchorloss}} + \dots \tag{1}$$

Vacuum sealing, selection of the operational frequency, and dynamically balanced design architectures are some of the mitigation strategies to overcome the energy losses. Low-frequency vacuum sealed devices, such as the MEMS Tuning Fork Resonator (TFR) and Quadruple Mass Gyroscope (QMG), are the examples that have demonstrated over 2M in Q-factor, close to their Thermo-Elastic Damping (TED) limit, [3,4]. On the other hand, we observed that other designs, within the same range of operational frequency and the same estimated TED, revealed orders of magnitude lower Q-factor than their predicted TED limits. In our analysis, the ratio of the measured Q-factor to the predicted Q-factor was orders of magnitude lower in devices with low frequency of separation between the operational and neighboring structural modes, Table I. Based on our observation of the energy distribution in the devices, the mechanical mode coupling has been identified as the dominating mechanism limiting the Q-factor. Analytical expressions were derived to capture the effect of fabrication imperfections on the abnormalities observed in the energy distribution among the operational and parasitic modes. The mechanical coupling between the in-plane and out-of-plane modes was modeled analytically and the drop in the Q-factor due to the coupling was estimated. Finally, we developed the design guidelines for attenuation of the effect of fabrication imperfections on the Q-factor of MEMS resonators and vibratory gyroscopes due to the phenomenon of modal coupling.

TABLE I
Parameters of devices for comparative study

Structure		
Design	CRG, [5]	QMG, [3]
Operational frequency	(5^{th} Mode) 2.31 kHz	(1^{st} Mode) 2.8 kHz
In-plane & out-of-plane frequency separation	0.13 kHz	3.6 kHz
TED Q-factor limit	1.8M	2.2M
Measured Q-factor (typical)	25.6k	1.8M
Q-factor ratio	1.4%	81.8%

II. Analytical Model

The mechanical modes in a lumped mass-spring structure can be represented through a second-order ODE:

$$[m_{ij}]\ddot{X} + [c_{ij}]\dot{X} + [k_{ij}]X = F(t)\hat{e}_o, \tag{2}$$

where X is a vector of displacement and $[m_{ij}]$, $[c_{ij}]$, and $[k_{ij}]$ are the inertial, damping, and stiffness matrices. $F(t)$ represents a harmonic force applied along the intended direction of displacement in the operational mode ($\hat{e}_o$) and at its natural frequency ($\sqrt{\lambda_o}$), calculated from a matrix A:

$$A = M^{-1}K \tag{3}$$

The eigenvectors (v_i) and eigenvalues (λ_i) of the matrix A are related to the modal energy distribution and the resonant frequency in each of the modes. In the case of an ideal fabrication process, the symmetry of the structure would result in a diagonal stiffness matrix and, consequently, in unit eigenvectors ($v_i = \hat{e}_i$). Unit eigenvectors would denote that all the energy is contained in the actuated mode. However, due to fabrication imperfections, the stiffness matrix $[k_{ij}]$ would consist of both diagonal and off-diagonal terms, which corresponds to the mechanical coupling among different modes of the system. In this case, the energy is distributed through the actuated operational mode and all neighboring modes ($v_i \neq \hat{e}_i$). In the case of the Concentric Ring Gyroscope (CRG) [5], an out-of-plane mode is the closest mode to the operational in-plane mode. Therefore, an abnormal energy distribution is observed, in which the proof-mass vibrates in both the in-plane and out-of-plane directions, while actuated in the operational in-plane mode.

The motion due to the mechanical mode coupling results in an energy dissipation through all the dominant energy loss mechanisms in the actuated operational mode and the neighboring modes. Therefore, the Q-factor of a device would be orders of magnitude lower than its Q-factor limit, in the operational mode calculated from Eqn. 1. The Q-factor due to mode coupling (Q_{MC}) can be represented as:

$$Q_{MC}^{-1} = \sum (a_i Q_i^{-1}), \quad 0 \leq a_i \leq 1, \qquad (4)$$

where Q_i is the total Q-factor in each of the operational and parasitic neighboring modes (Eqn. 1), and a_i determines the contribution of each mode to the energy loss. The actual value depends on the frequency separation and the mechanical coupling that exists between the parasitic and operational modes ($k_{ij} : i \neq j$).

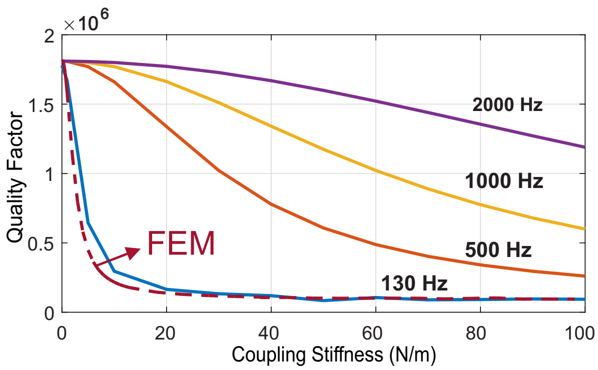


Fig. 1. Calculated quality factor due to mode coupling versus the mode coupling stiffness ($k_{ij} : i \neq j$), considering frequency separations 130, 500, 1000, and 2000 Hz between the in-plane and out-of-plane modes. Analytically, we observed a higher impact of fabrication imperfections on Q-factor in lower modal frequency separations due to mode coupling.

Using the expression shown in Eqn. 2, the Q-factor for the CRG design was calculated considering the mechanical coupling between the in-plane and out-of-plane modes, for different assumptions of modal frequency separation, Figure 1. The TED Q-factors of the in-plane and out-of-plane modes were used as the damping parameters and were calculated using COMSOL Multiphysics to be 1.8M and 50k, respectively. The energy dissipation through the anchors was considered to be negligible due to the reduced absolute magnitude of reaction forces and stresses at the anchor in the studied low-frequency devices. The mode coupling stiffness ($k_{ij} : i \neq j$) quantifies asymmetries of the structure due to fabrication imperfections and based on experimental results were chosen to be 0-100 N/m.

III. Experimental Results

Two CRG and QMG designs were experimentally evaluated. In the experiment, the in-plane amplitude of vibration was electrostatically measured and controlled, both in amplitude and frequency. The out-of-plane motion was measured using a Laser Doppler Vibrometer (LDV). The CRG, as a design with low frequency of separation between the out-of-plane and in-plane modes, showed 5000 times higher energy ratio between the out-of-plane to the in-plane modes, compared to the QMG with the corresponding ratio of 0.28 ppm. It confirmed a higher mechanical mode coupling in the CRG structure and explained a much lower Q-factor in these devices compared to high frequency mode-separated devices, such as the QMG, Figure 2.

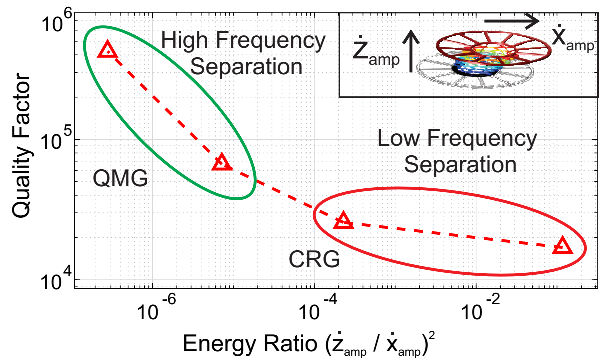


Fig. 2. Experimental results of the measured energy ratio of out-of-plane to in-plane vibration for two QMGs and two CRGs, with parameters shown in Table I. Mode coupling stiffnesses of 20-60 N/m were calculated for the structures.

IV. Conclusion

In this paper, we identified the phenomenon of mechanical mode coupling as a dominant energy loss mechanism in low frequency vibratory devices. Finite element analysis and analytical modeling were performed to estimate the quality factor due to the mode coupling. Both experimental and modeling results were presented, confirming a higher effect of fabrication imperfections on the Q-factor of devices with low-frequency separation between modes. The increase in the frequency separation between the operational and neighboring modes was emphasized as a criteria for achieving and maintaining the high Q-factor despite fabrication imperfections.

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