Multi-Degree-of-Freedom MEMS Coriolis Vibratory Gyroscopes Designed for Dynamic Range, Robustness, and Sensitivity

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Abstract

An overview of multi-mass solutions for MEMS Coriolis vibratory gyroscopes is presented in this paper. The advantages and challenges associated with increasing the number of Degrees-of- Freedom (DOF) of the mechanical element are discussed.

1. Introduction

Conceptually, a single proof-mass is required to measure the Coriolis-acceleration-induced angular rate signal along a single axis. In recent years, however, a variety of multi-mass solutions emerged, offering advantages, such as a dynamic structural balance, Fig. 1.1(a), an increased bandwidth of detection, Fig. 1.1(b), and a dynamic amplification of response, Fig. 1.1(c).

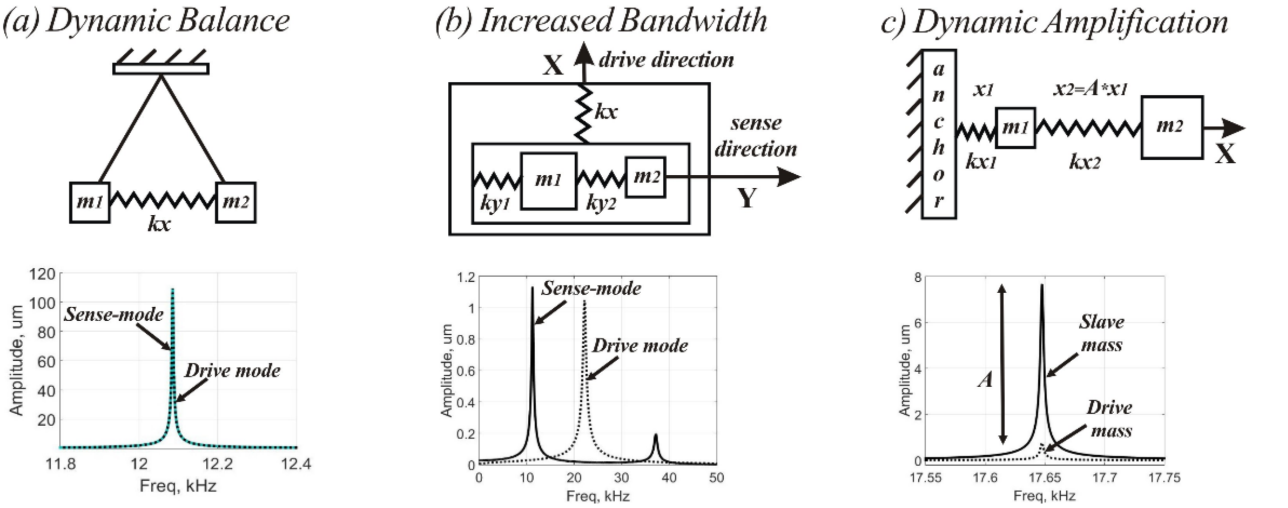

Figure 1. Examples of multi-mass gyroscopes: (a) dynamically balanced two-mass system, (b) dual-mass system with 2-DOF sense-mode oscillator; (c) dynamically amplified system.

Dynamically balanced systems, Fig. 1.1(a), such as, for example, a Dual Foucault Pendulum (DFP) gyroscope, [1], utilized two or more dynamically equivalent, mechanically coupled proof-masses, oscillating in the anti-phase motion, for improved vibration immunity and anchor loss mitigation, resulting in the ultra-high quality factor. The concepts of a dynamic balance for anchor loss mitigation and a common-mode rejection of shock and vibration are employed in the design of the Tuning Fork (TF) Gyroscope, [2], where two dynamically equivalent, mechanically coupled proof-masses are utilized. A similar principle is employed in the design of a Quadruple Mass Gyroscope (QMG), [3], where the structural element is formed by four mechanically coupled proof masses, and thus enabling a dynamic balance of forces and moments in drive and sense modes, as opposed to a dynamic balance of forces in a single-axis TF architecture.

In [4], C. Acar et al. introduced an increased bandwidth, inherently robust dual-mass gyroscope. The mechanical element was comprised of a two DOF sense mode oscillator, formed by two interconnected masses, Fig. 1.1(b). The device was operated in a flat region of the sense-mode response curve, where the amplitude and phase of the response are insensitive to environmental fluctuations. For example, it was experimentally demonstrated in [4] that a temperature variation from 25 to 75ºC resulted in only 1.62% change in the output of a wide-bandwidth gyroscope, which was 12.2 times smaller as compared to a conventional 1-DOF sense mode gyroscope.

Another example of a multi-mass solution is a dual-mass dynamically amplified system, Fig. 1.1.(c), where an increased number of DOF results in dynamic amplification of motion and improved sensitivity, [5]-[6]. In a dynamically amplified gyroscope, the first, the “drive mass", is actively driven to oscillate at a small amplitude of motion, in a linear operational regime. Meanwhile, the mechanically coupled “slave mass" is used for sensing the Coriolis signal. The amplitude of motion of the “slave mass" is dynamically amplified, resulting in an increased scale factor of the device.

This paper offers a review of different multi-degree of freedom gyroscope concepts. The advantages of such systems are analyzed, and potential challenges are discussed.

2. Single-axis one-mass Classical Gyroscope

The operation principle of MEMS vibratory gyroscopes is based on a transfer of energy between two vibration modes, that occurs due to Coriolis acceleration under applied angular-rate input. In one simple implementation of a single-axis one-mass gyroscope, the proof-mass is suspended above the substrate by four flexible beams.

The vibrating mass MEMS gyroscope has two orthogonal mechanical excitation modes along which the mass can move, Fig. 2. In rate-measuring mode, the primary mode is generally excited along X (drive axis) by an external sinusoidal electrostatic or electromagnetic force. The secondary mode along Y (sense axis) is induced by the Coriolis force in presence of inertial rotation. The energy transfer between the drive and the sense- modes is proportional to the applied angular rate input.

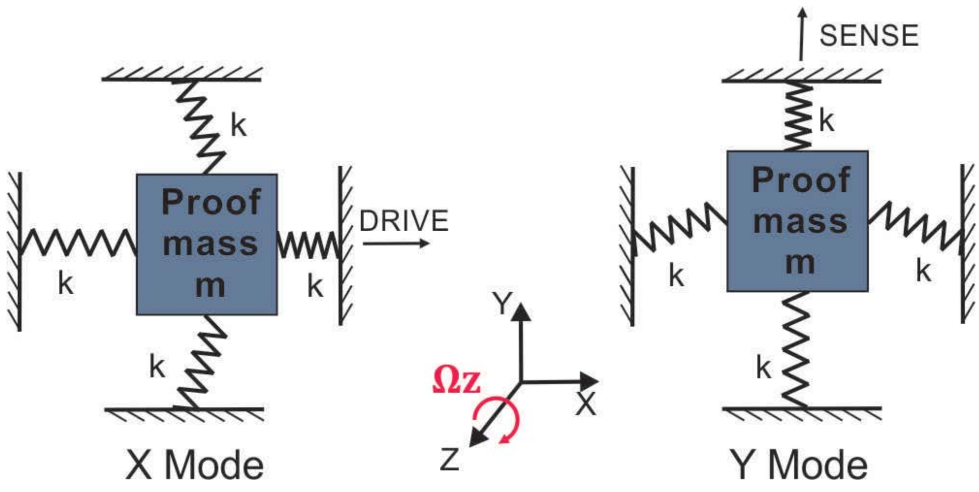

Figure 2. Single-axis one-mass gyroscope concept.
The rotation-induced Coriolis force causes energy transfer between the drive (X-mode) and the sense (Y-mode) proportional to the angular rate input.

To maximize the device scale factor, it is desired to match the resonant frequencies of the drive and sense mode oscillations. This is typically achieved by electrostatic tuning or permanent trimming of the mechanical element. Mode-matching, however, results in extreme sensitivity to variations in oscillatory system parameters that shift the natural frequencies and introduce errors. Alternatively, the sense-mode is designed to be slightly shifted in resonance frequency from the drive-mode, thus enabling improved robustness to environmental fluctuations, while intentionally sacrificing gain and sensitivity.

Energy loss in the system is another factor contributing to gyroscope performance. High quality factor is necessary for maximized sensitivity and reduced noise. In one-mass gyroscopes, anchor loss is one of the major energy loss mechanisms. Anchor loss occurs due to externally propagating stress waves through the substrate, [30]. As a single-mass gyroscope vibrates in drive- or sense-mode, the vector of momentum oscillates along a single line of motion, causing deflection of the substrate and producing stress waves that expand outward from the structure. This form of energy loss can often be dominating in a single-mass gyroscope, limiting its performance.

Acceleration sensitivity can be a major challenge for high-Q gyroscopes. The mode shapes of a single-mass gyroscope are not balanced in force and in momentum, leading to both high energy loss through the substrate and high sensitivity to common-mode acceleration.

To address the described challenges, several multi-degree-of systems have emerged in recent years.

2. Multi-Degree-of-Freedom Systems for Dynamic Balance

2.1. Tuning Fork Gyroscope

The classic example of a dynamically balanced design of a Coriolis vibratory gyroscope is a Tuning Fork (TF) Gyroscope, reported by Draper Laboratory in 1993, [2]. The mechanical element of the gyroscope was comprised of two proof-masses, electrostatically excited to vibrate along the drive direction in anti-phase, Fig. 3. A set of comb drives served for electrostatic excitation. Angular rate applied along the Y-axis perpendicular to the drive mode, excites the out-of-plane anti-phase vibration of the two proof-masses. The sense response in the out-of-plane rocking mode was detected electrostatically using bottom electrodes [2].

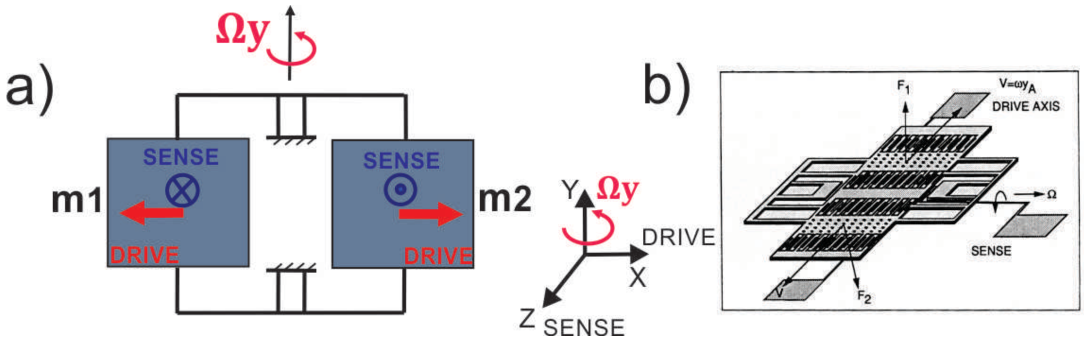

Figure 3. (a) lumped mass model of Y-axis Tuning Fork Gyro,
(b) schematic drawing of the Draper Laboratory Comb-Drive Tuning Fork Gyro, [2].

Any external acceleration creates a static deflection of the two proof-masses. The anti-phase operation of TF gyroscope creates a first order rejection of common-mode acceleration. Meanwhile, the substrate energy loss is mitigated through the use of anti-phase modes. The TF structure minimizes the anchor losses as stresses from the two arms cancel at the mounting locations.

The in-plain drive mode of the TF gyroscope is dynamically balanced in forces and momentum. However, in the sense-mode, the two proof-masses oscillating out-of-plane in anti-phase, create momentum around the sensitivity axis. Hence, a full dynamic balance of momentum cannot be achieved, by design.

2.2. Dual Foucault Pendulum (DFP) Gyroscope

Dual Foucault Pendulum (DFP) Gyroscope, [7] is an example of a minimum realization of a lumped mass gyroscope configuration that can provide a fully dynamically balanced system, Fig. 4(a). Core of the gyroscope architecture is two mechanically coupled and dynamically equivalent proof masses, oscillating in anti-phase motion.

Dynamic balance is obtained by aligning the center of masses of each proof mass. This allows the center of mass of the system to remain stationary during oscillation, causing the net force and torques generated by the combined system to be zero, Fig. 4(b). Unlike a conventional tuning fork gyroscope, the force and torque balances are obtained on both x and y modes, providing immunity to vibration and shock as well as anchor loss mitigation.

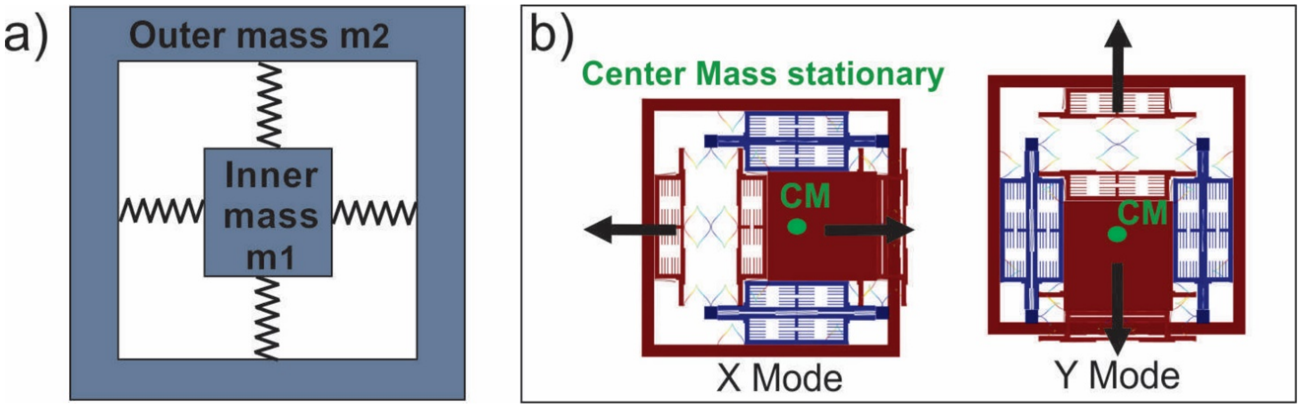

Figure 4. (a) minimum realization of a lumped mass dynamically balanced system,
(b) Dual Foucault Pendulum (DFP) Gyroscope X-Y symmetric anti-phase operation, [1], [7].

In the implementation described in [1] and [7], dynamic equivalence of the two proof masses is achieved by using identical suspension elements and shuttle assemblies, while designing the two proof masses to have equal masses. This results in same resonance frequencies for individual proof masses, which is also supported by mechanical coupling of the two proof-masses. This mechanical coupling is achieved via "weak springs" between shuttle assemblies of inner and outer proof masses, which synchronizes phases of the proof masses. Electrostatic transduction is provided by arrays of parallel plates located on the shuttle assemblies.

Ring-down characterization of the mechanical element at a vacuum level of 5 µTorr showed an energy decay time constant (τ) of 30 s at 3.2 kHz, which corresponds to Q-factor over 300,000 on both modes. Due to the high-Q degenerate mode operation, large modal mass, and large nominal capacitance angle random walk (ARW) as low as 0.003 $°/\sqrt{h}$ and an in-run bias stability of 0.27 $°/h$ was demonstrated in open loop operation without temperature control or compensation.

2.3. Quadruple-Mass Gyroscope

Another well-known example of a fully dynamically balanced multi-degree-of freedom structure is a Quadruple-Mass Gyroscope. The concept of the Quadruple-Mass Gyroscope was introduced in [3].

The design of Quadruple-Mass Gyroscope utilizes two symmetric single-axis tuning fork structures, placed parallel to one another and coupled together with additional linear flexures and antiphase lever mechanisms. The design symmetry along both axes is enabled by identical proof-mass dimensions, equivalent suspension flexures, levers, and symmetric electrode architectures. Operation of the QMG is illustrated in Fig. 4. This X−Y symmetric four-mass system provides the structure with two antiphase dynamically balanced modes of mechanical vibration at a single operational frequency.

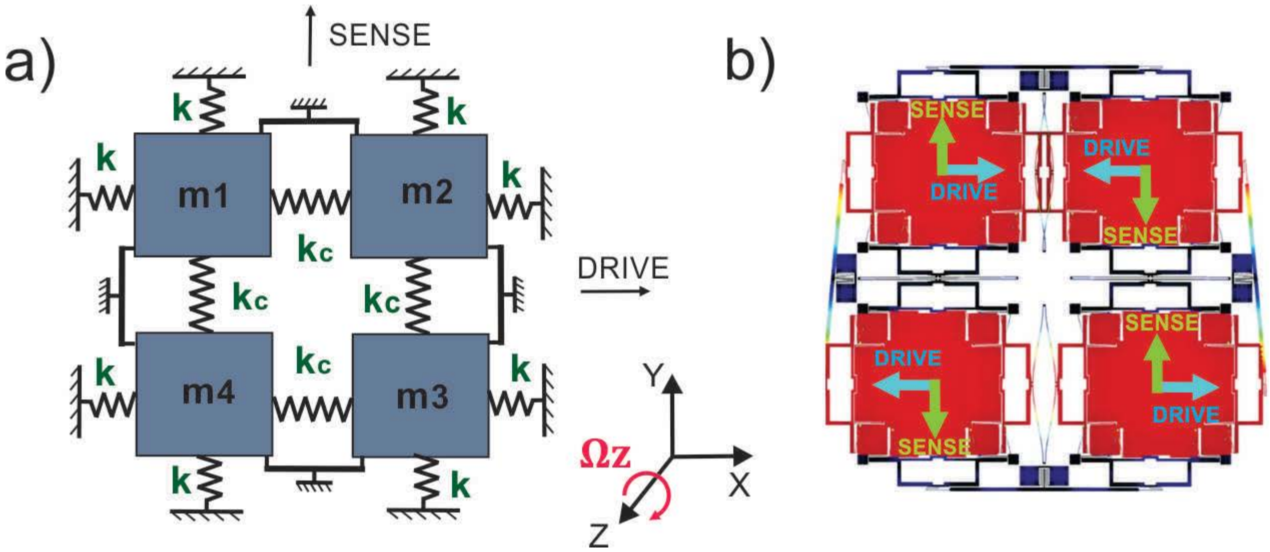

Figure 4. Quadruple-Mass Gyroscope (a) four-mass coupled system dynamically balanced in force and momentum, (b) X-Y symmetric anti-phase operation, [3].

The balanced antiphase design of QMG aims to minimize frequency and damping mismatches and to maximize the Q-factor. A Si QMG with the quality factor higher than 2 Million at drive-mode resonance frequency of 1.68 kHz was reported in [8]. QMG with near-navigational grade performance noise (Angle Random Walk (ARW) of 0.02 $°/\sqrt{h}$) was demonstrated in [9].

QMG architecture expands the structural advantages of the dual-mass tuning fork design. Unlike conventional tuning fork devices, QMG is fully dynamically balanced in forces and momentum and provides mechanical rejection of external vibrations and mechanical shocks along both the drive and sense axes.

Additional important advantage of a QMG design is compatibility with a rate integrating mode of operation enabled by low energy dissipation and isotropy of both the resonant frequency and damping.

The advantages of the fully balanced QMG system has attracted a lot of research interest. The QMG-type gyroscope architectures were explored recently by a number of research groups, including [10], [11].

3. Multi-Degree-of-Freedom Systems for Robustness

3.1. Approach 1. Operation in Valley

C. Acar et al. explored a multi-degree-of-freedom wide-bandwidth gyroscope design concept that provides inherent robustness against structural and environmental parameter variations, [4]. The described “mass-in-mass” gyroscope dynamical system consists of a 2-DOF sense-mode oscillator and a 1-DOF drive-mode oscillator, formed by two mechanically coupled proof masses, Fig. 5. The first mass is excited to oscillate along the X-axis in the drive direction. The second mass is constrained in the drive direction with respect to the first mass. In the drive direction, the two proof-masses oscillate together, forming a resonant 1-DOF oscillator.

The 2-DOF sense-mode oscillator provides a frequency response with two resonant peaks and a flat region between the peaks. The device is operated in the valley between the two peaks of the sense-mode response curve, where the gain is less sensitive to variations in the natural frequencies and damping. The proposed approach allows to achieve reduced sensitivity to structural, thermal parameter fluctuations and changes in damping.

Improved robustness of the wide-bandwidth design concept was experimentally verified in [4]. Operation in a 200 Hz wide flat valley of the 2-DOF sense-mode oscillator was experimentally demonstrated. The authors showed that the sense-mode response in the flat operating region was inherently insensitive to pressure, temperature, and dc bias variations. The maximum amplitude variation in the flat operating region was observed to be less than 2% for the 50°C variation in temperature. The change in the response amplitude in the flat operating region was insignificant under the change in pressure level between 5 and 30 Torr and under variations of dc bias voltages in the range from 18 to 21V.

This multi-DOF design concept is expected to lead to reliable, robust, and high performance-angular-rate sensors with low-production costs and high yields, ideal for demanding automotive environment.

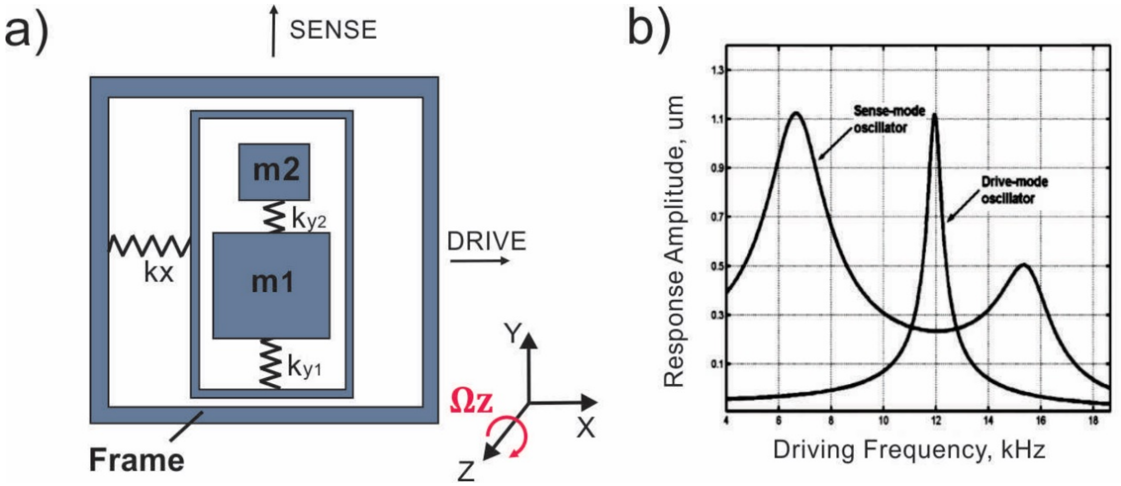

Fig.5. Concept of 3-DOF gyroscope with 2-DOF sense mode: a) Lumped mass-spring model, b) Frequency response showing operation in a valley between two peaks of a sense-mode oscillator.

3.2. Approach 2. Distributed mass concept

The Distributed-Mass Gyroscope (DMG) concept also aims to improve robustness by utilizing multi-degree-of freedom system. The approach is based on employing multiple drive-mode oscillators with incrementally spaced resonance frequencies, Fig.6. A wide-bandwidth levelled region in the drive-mode response allows for improved robustness against structural and thermal parameter variations.

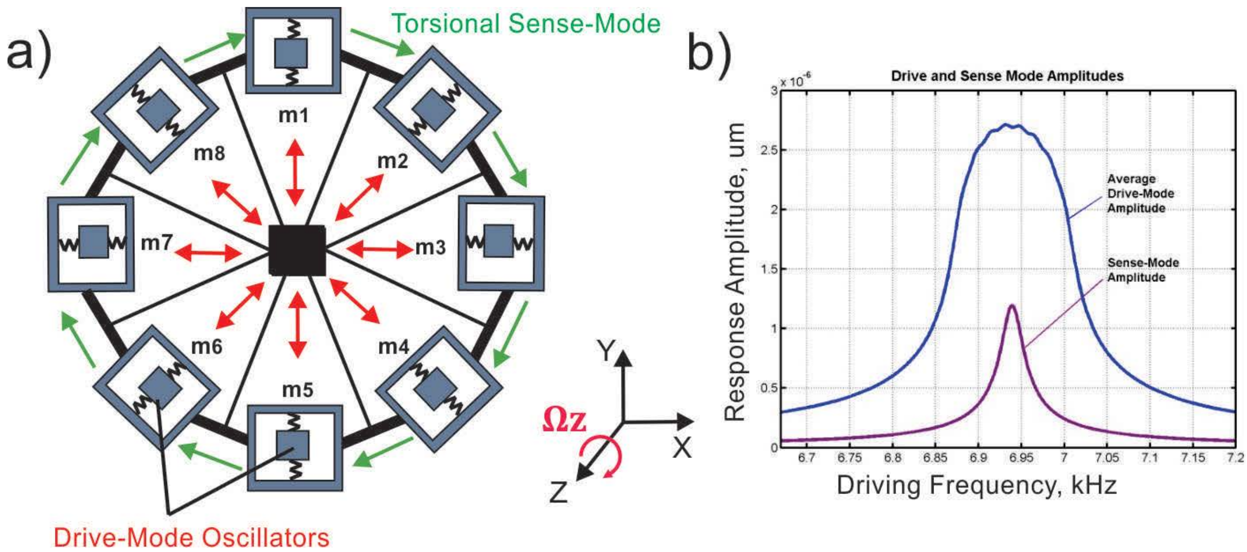

Fig.6. Concept of distributed mass gyroscope: a) Lumped mass-spring model, b) multiple oscillators enable wide-bandwidth response in the drive-mode, [13].

The design concept presented in [12] is based on employing multiple drive-mode oscillators, distributed symmetrically around the center of a supporting frame. The distributed drive-mode oscillators are driven in-phase along the drive axes normal to the tangents of the supporting frame. In the presence of an angular rotation rate about the Z-axis, the Coriolis forces are induced on each proof mass orthogonal to each drive-mode oscillation direction. The net Coriolis torque excites the supporting frame into torsional oscillations about the z-axis, which are detected by sensing capacitors for angular rate measurement. The approach was experimentally verified in [13]. A levelled 600Hz wide-bandwidth drive-mode response with a maximum 17.2% variation in amplitude was achieved, showing that the natural frequency scatter due to imperfections could be utilized to provide the required frequency spacing for wide-bandwidth operation.

One challenge of the approach is random distribution of the resonant frequencies of the drive-mode oscillator, due to fabrication imperfections. In [13], electrostatic tuning of the oscillators was performed to reduce the split between the drive-mode resonance frequencies so that the resonators could be excited together, to jointly generate a resultant Coriolis torque. Utilizing higher resolution fabrication technologies, the random scatter could be decreased further, and the oscillators could be ultimately designed with incrementally spaced resonant frequencies to provide the required uniform spacing.

The discussed multi-degree-of-freedom designs allow to widen the operation frequency range of the gyroscope drive-mode to achieve improved robustness. However, this is achieved at the expense of sacrificing in the response amplitude. In distributed-mass gyroscope, an optimal compromise between amplitude of the response and bandwidth (affecting sensitivity and robustness, respectively) is obtained by selecting the frequency increments of the drive-mode oscillators. In case of a “mass-in-mass” wide-bandwidth gyroscope design concept, the trade-offs between gain of the response (for higher sensitivity) and the system bandwidth (for increased robustness) are achieved by optimizing parameters of the mechanical system, including the proof-masses ratio and stiffness of suspension.

Multi-degree-of-freedom MEMS gyroscopes employing a wide-bandwidth design concepts are shown to be inherently robust against structural and environmental parameter variations. In such systems, the disturbance-rejection capability is achieved through design of gyroscope dynamical system instead of active control and compensation techniques. MEMS gyroscopes of this class could potentially provide reliable, robust and high performance angular-rate measurements leading to a wide range of applications including dynamic vehicle control and automotive safety systems.

3. Multi-Degree-of-Freedom Systems for Amplitude Amplification

In dynamically amplified dual-mass MEMS gyroscope the increase in structural degrees of freedom is used to improve sensitivity, linearity, and to reduce drift, [14], [15]. The gyroscope design utilizes the principle for motion amplification in a coupled multi-degree-of-freedom system. In one implementation [16], the gyroscope is comprised of two proof-masses interconnected by means of concentric ring suspension and driven to oscillate in the in-phase motion, Fig. 7.

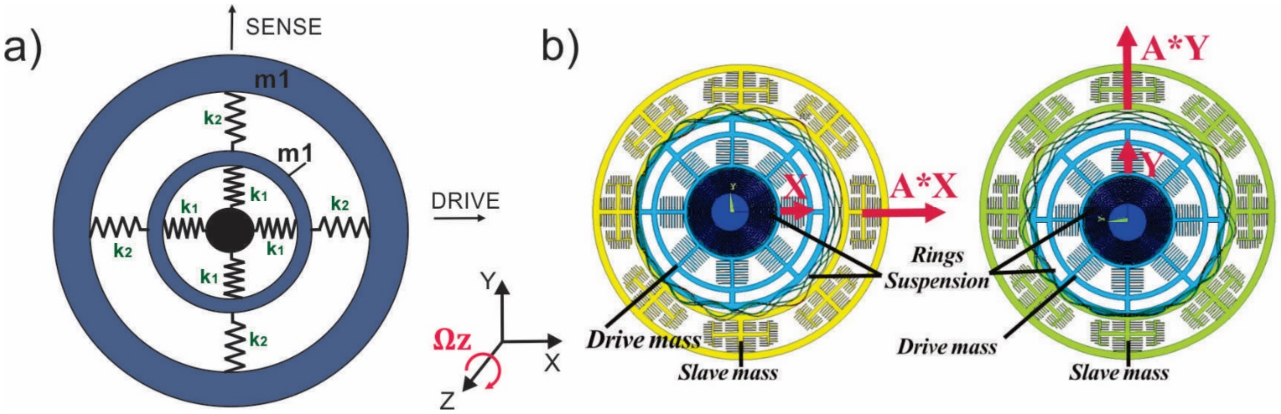

Fig.7. Micro-photograph of a fabricated prototype of the dynamically amplified dual-mass gyroscope.

The first “drive mass" is actively driven to oscillate at a small amplitude of motion. Meanwhile, the mechanically coupled “slave mass" is used for sensing the Coriolis signal. The amplitude of motion of the “slave mass" is dynamically amplified. Prototypes of dynamically amplified gyroscope with amplification factor up to 10x were reported in [16].

The main advantage of the described architecture is that it allows for linear control of the small-amplitude vibrations of a driving mass, while enabling high scale factor of the sensing mass. In addition, the axisymmetric structure of dynamically amplified gyroscope allows for rate and rate measuring modes of operation, [16].

A comparative study of a conventional single-mass gyroscope and a dynamically amplified dual-mass gyroscope of a similar footprint showed over 3x improvement in sensitivity of a dual-mass sensor and superior performance in terms of bias stability and Angle Random Walk (ARW), [17].

3. Conclusion

There are several different goals a gyroscope designer might pursue while choosing to increase the number of degrees-of-freedom in the system.

In mechanically coupled fully dynamically balanced multi-DOF systems, anchor losses though the substrate can be mitigated, enabling ultra-high Q-factor devices with low angle random walk and low in-run bias stability. In addition, common-mode rejection of accelerations also contributes to reduction of the gyroscope noise and improvement of the scale factor linearity. Noise characteristics of these devices showed a potential for achieving the navigation grade accuracy (QMG with ARW <0.02 deg/rt-hr, [4]). With further improvements in structural symmetry, this type of devices may enable silicon micromachined devices for inertial guidance applications.

Multi-DOF robust MEMS gyroscopes designs provide reduced sensitivity to structural and environmental parameter variations. This class of devices require less demanding active compensation schemes and are expected to relax requirements for fabrication process accuracy and packaging tolerances. Inherent robustness, however, is generally achieved at the cost of reduction in the gyroscope scale-factor. An optimum balance between the amplitude of the response and bandwidth affecting robustness is necessary to yield reliable, robust and high performance angular-rate measurements.

Dynamic amplification of motion in multi-DOF structures is a promising concept for implementation of high-performance MEMS gyroscopes. In this type of devices high amplitudes of vibration can be achieved without sacrificing linearity of the drive system enabling improved scale factor and reduced drift.

Along with many advantages of the multi-mass systems, the structural symmetry is a challenge. The symmetric mechanical element is necessary for the mode-matched devices, and a method of compensation for fabrication imperfections is required. The increased number of DOF of a mechanical element results in a more complex form of the structural stiffness matrix, as compared to a conventional single-mass gyroscope. Hence, more complex techniques must be considered for tuning of operational modes in the case of a non-ideal multi-mass structural element.

Different methods for precision tuning of operational modes can be exploited for multi-degree-of-freedom systems, including electrostatic tuning and permanent trimming of the mechanical element.

Mechanical trimming techniques are based on a permanent modification of the structural element by means of selective adding or removing of material. Such methods can employ, for example, laser ablation, [18]-[19], focused ion beam, [20], and selective deposition, [21]-[22]. The main advantage of the mechanical trimming method is a permanent tuning of vibration modes, which stays in place after device turn-off, turn-on. This enables minimization of voltages necessary for subsequent electrostatic tuning. The passive tuning method, however, is mainly limited to off-line calibration in a laboratory environment.

Mechanical trimming was successfully employed in [18]-[22]. In these publications, however, the authors mainly deal with a single-mass structures or wine-glass structures utilizing single suspension formed by a series of concentric rings. In both cases, the perturbation analysis takes into consideration a 2x2 electrostatic stiffness matrix. In cases of multi-mass gyroscopes, described in this article, increasing the number of DOF of the mechanical element results in a higher order system's stiffness matrix. Hence, new algorithms have to be developed for precision trimming.

As opposed to mechanical trimming, active tuning techniques do not introduce a permanent modification of the structure and can be implemented in real-time after the sensor is packaged, including during an on-line in-field calibration. A number of active tuning approaches has been reported in literature, including thermal tuning, [23], feedback control methods, [24]-[26].

Electrostatic tuning, which employs electrostatic force gradients to selectively modify the effective stiffness of a certain operational mode, is another common active-tuning technique for the gyroscope mode-matching. Precision electrostatic tuning algorithms were successfully employed for 2-DOF systems, [27]-[29] and multi-DOF systems, [16].

Despite the increased complexity of the mechanical element, gyroscopes based on multi-DOF systems have a wide range of advantages and the choice of a specific conceptual design is defined by the target application. Wide-bandwidth robust gyroscopes, for example, may meet specific requirements of automotive industry. On the other hand, high-sensitivity, low-noise MEMS gyroscopes are suitable for integration within a high-end MEMS inertial measurement unit. Shifting complexity towards design of the multi-DOF mechanical element, along with utilization of advanced low-loss materials (such as fused quartz and silicon carbide) is a promising approach towards realization of the highly accurate MEMS inertial navigation system.

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