TEMPERATURE SENSITIVITY OF ANGULAR GAIN IN MICRO RATE-INTEGRATING GYROSCOPES (MRIG)

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ABSTRACT

This paper provides a study on temperature sensitivity of angular gain and its correlation with effectiveness of quadrature control in Micro-Rate-Integrating Gyroscopes (MRIG). As a result of Temperature Coefficient of Frequency (TCF)-mismatch in the mechanical element of MRIG, temperature variations affect the sensitivity in direct angle measurements (i.e., angular gain). In our study using a Dual Foucault Pendulum (DFP) micro-gyroscope as a testbed, we experimentally demonstrated that Angular-Gain-Temperature-Sensitivity (AGTS) is coupled to effectiveness of the quadrature feedback controller that is implemented to compensate for anisoelasticity. For the DFP gyroscope, with a TCF-mismatch on the order of 603 ppb/°C, the angular gain variation was measured to be on the order of 820 ppm for temperatures ranging from 17 °C to 45 °C. We implemented a Dynamic Quadrature Compensator (DQC), which reduced AGTS by 63-times, down to 13 ppm. The results of this paper demonstrated feasibility of using MEMS devices as MRIG and provided a mitigation strategy to achieve an increased angular gain stability, despite a TCF-mismatch in the sensing element.

KEYWORDS

MEMS, gyroscopes, angular gain, whole-angle, TCF, angular drift.

INTRODUCTION

Type I Coriolis Vibratory Gyroscopes (CVG) can be utilized for angular rate and direct angle measurements [1]. To instrument a CVG for direct angle measurement, a Whole-Angle (WA) control architecture needs to be employed [2]. In the WA mode of operation, the mechanical resonator functions as an angular rate-integrating sensor that can theoretically measure rotations with no limit on the angular velocity.

In an ideal Rate-Integrating Gyroscope (RIG) with a symmetric structure, the resonator is excited to oscillate along a straight line. In the presence of rotation, the oscillation pattern "precesses," as illustrated in Fig. 1(a). The angle of precession (θ) is equal to the time-integration of the angular rate of rotation (Ω) multiplied by the Angular Gain (AG) factor (k). In Micro-Rate-Integrating Gyroscopes (MRIG), fabrication imperfections and material anisotropy adversely affect the symmetry of the structure, introducing anisodamping and anisoelasticity in the mechanical structure. It has been reported that anisodamping results in a reduction of the AG and angle-dependent bias errors in MRIG instrumentation [3]. On the other hand, anisoelasticity results in ellipticity of the oscillation pattern, shown in Fig. 1(b), which in turn reduces the AG.

Assuming that the anisodamping and anisoelasticity are constant, the Effective Angular Gain (EAG), denoted by k_e, can be characterized and used to estimate the input angle of rotation. However, as reported in [4], anisoelasticity or anisodamping variations during the operation of an MRIG would change the EAG and cause angular drifts in measurements. Angular drifts due to variations of EAG are unbounded and grow over the time of operation. Therefore, it is essential to minimize the instability of EAG to improve the accuracy of direct angle measurements using MRIG.

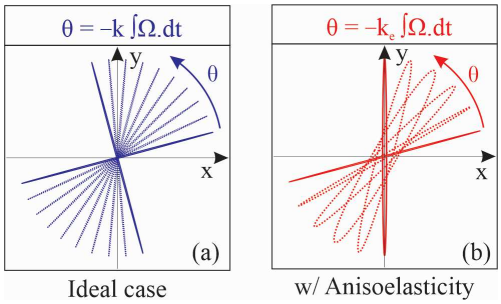

Figure 1: Evolution of oscillation pattern in MRIG in presence of rotation. Anisoelasticity in micro-fabricated gyroscopes results in ellipticity of the oscillation pattern that reduces the angular gain (i.e., $k > k_e$).

When rotation with a high angular velocity is applied to MRIG, the gyroscope provides a better resolution for angle measurements. Therefore, MRIG is especially beneficial for use in scenarios in which rotations with high angular velocity are measured. Alternatively, Virtual Carouseling (VC) can be implemented to offset the rate of precession ($\dot{\theta}$), to improve the resolution of angle measurements [3]. At a relatively high rate of precession, the effect of anisodamping on EAG becomes negligible [5]. Therefore, in this paper, we focused on the effect of anisoelasticity variations on the angular gain.

A mismatch in Temperature Coefficient of Frequency (TCF) of the operational modes is the primary mechanism causing anisoelasticity variations during operation. TCF of MRIG along the X and Y axes are typically non-identical [4], and due to a mismatch in TCF, the frequency split changes with temperature. Although a feedback controller is often utilized in the WA control architecture to compensate for ellipticity, if ellipticity is not fully compensated, Angular-Gain-Temperature-Sensitivity (AGTS) becomes inevitable [5].

We studied AGTS caused by TCF-mismatch in MRIG and its correlation with effectiveness of quadrature compensation in the WA control.

MATHEMATICAL MODEL

Based on a reduced-order model of MRIG reported in [2], the following differential equation describes the effect of anisoelasticity on precession:

$$\dot{\theta} \approx -k\Omega + \frac{1}{2}\Delta\omega\frac{Q}{E}\cos 2(\theta - \theta_\omega), \qquad (1)$$

where parameters $\Delta\omega$, θ_ω, Q, and E denote the frequency split, misalignment in the principal axes of elasticity, quadrature, and energy in MRIG. In the WA control architecture, ellipticity in the oscillation pattern is quantified using the quadrature variable (Q), and it is used in a feedback loop to apply quadrature compensation forces (f_{qc}) to reduce ellipticity.

The following differential equation describes the evolution of quadrature in presence of anisoelasticity [2]

$$\dot{Q} \approx -2\frac{Q}{\tau} + u_\omega + \frac{\sqrt{E}}{\omega} f_{qc}, \qquad (2)$$

where anisoelasticity is considered as a perturbation in the form of

$$u_\omega = -\Delta\omega \, \mathrm{E} \sin 2(\theta - \theta_\omega), \qquad (3)$$

and τ, ω, and f_{qc} denote the average energy decay time constant, the average resonant frequency, and the quadrature control force, respectively. From equations (2) and (3), it can be concluded that if quadrature control is not utilized ($f_{qc} = 0$), anisoelasticity would result in a large amount of quadrature, and in turn, affect the free precession.

Conventionally, a Proportional-Integral (PI) feedback loop is used for compensation of quadrature [2], illustrated in Fig. 2. Since during precession, anisoelasticity acts as a harmonic time-varying perturbation, Eq. (3), a PI controller cannot fully eliminate ellipticity. As a result of the residual quadrature, anisoelasticity affects the free precession and the Effective Angular Gain (EAG) is reduced [5]. In this case, the anisoelasticity-induced reduction of EAG is quadratically dependent on the frequency split ($\Delta\omega$). If frequency split changes during operation, for example due to temperature variations, so does the EAG. By applying a quadrature control that fully eliminates quadrature ($Q = 0$), the effect of anisoelasticity on precession and the dependency between EAG and frequency split would diminish.

The quadrature control proposed in [6] has been reported to fully eliminate quadrature. As illustrated in Fig. 2, by utilizing a feedback loop that applies an angle-dependent compensation force along with the quadrature PI control, we can fully compensate for anisoelasticity at the source. Through a sequence of demodulation, integration, and modulation of residual quadrature based on the pattern angle (θ), the quadrature control force converges to a harmonic force with an amplitude equal to the anisoelasticity perturbation (u_ω) and a phase relation that would destructively eliminate anisoelasticity. We refer to this control architecture as a Dynamic Quadrature Compensator (DQC). Mathematical background on stability and convergence of such control architecture can be found in [7].

To experimentally verify this, we implemented a WA control with a Dual Foucault Pendulum (DFP) micro-gyroscope [8] and characterized EAG at different temperatures. We experimentally demonstrated the improvement of the Angular-Gain-Temperature-Sensitivity (AGTS) through applying the DQC and eliminating ellipticity.

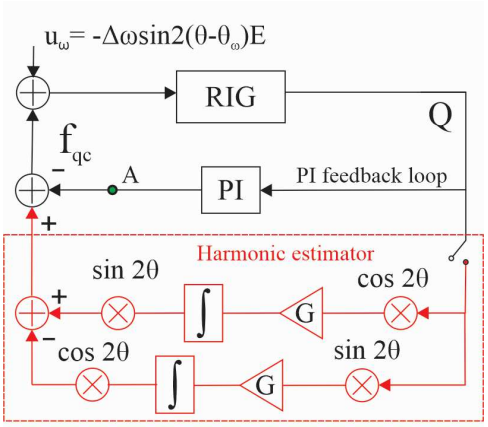

Figure 2: Block diagram of a Dynamic Quadrature Compensator (DQC), comprised of a PI feedback loop and a harmonic estimator. The harmonic estimator utilizes residual quadrature (Q) to estimate and cancel the harmonic perturbation due to anisoelasticity (u_ω).

EXPERIMENTAL RESULTS

To demonstrate the correlation between AGTS and TCF-mismatch, a DFP gyroscope, illustrated in Fig. 3, was instrumented for direct angle measurements using the WA control. Design parameters of the DFP gyroscope can be found in [9]. For compensation of anisoelasticity, we explored two control architectures, a PI feedback loop and a DQC proposed in [2] and [6], respectively.

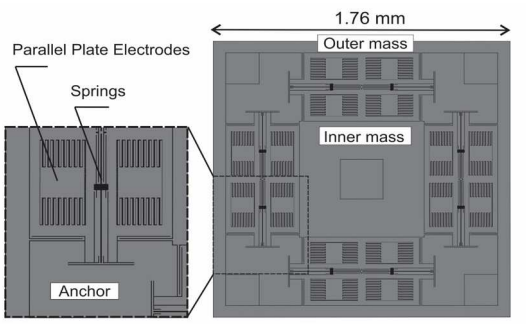

Figure 3: Design of an axisymmetric Dual Foucault Pendulum (DFP) micro-gyroscope [8]. We instrumented the DFP gyroscope for direct angle measurement to investigate the temperature sensitivity of angular gain.

The WA control was implemented using the Real-Time Kit (RTK) on a Zurich instrument HF2Li lockin-amplifier. By physically rotating the DFP at 360 Degrees-Per-Second (DPS), we were able to demonstrate precession, shown in Fig. 4.

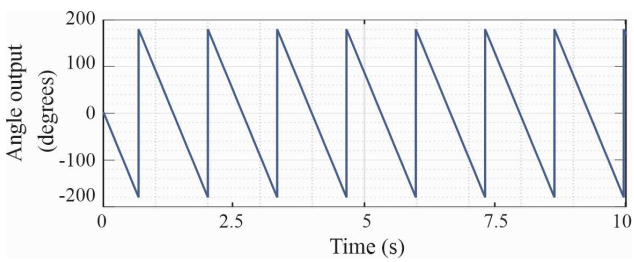

Figure 4: Demonstration of precession during 360 DPS of rotation applied to the DFP MRIG.

Effectiveness of Quadrature Control

In this section, we experimentally compared effectiveness of quadrature control architectures. In the experiment, the frequency split of the DFP was electrostatically reduced from 114 Hz to 302 mHz. We first applied a PI feedback loop as the quadrature control. Twelve seconds into the operation, we engaged the harmonic estimator as a part of the DQC control, Fig. 2. The variations of quadrature and the output of the PI feedback loop (node A in Fig. 2) are shown in Fig. 5.

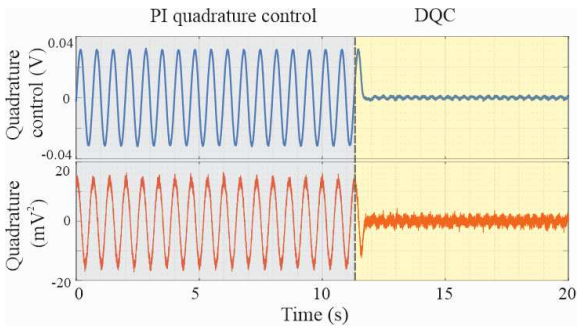

Figure 5: Experimental measurement of quadrature and output of the PI quadrature control are shown for two different quadrature control architectures.

By plotting the variation of quadrature as a function of the pattern angle (θ), Fig. 6, we were able to evaluate the effectiveness of the quadrature controls. By not utilizing a quadrature control, the ellipticity (quadrature) was observed to change harmonically with a large amplitude. The PI quadrature control reduced quadrature by more than 10-times. By applying the DQC, we were able to fully eliminate ellipticity in the DFP MRIG, Fig. 6.

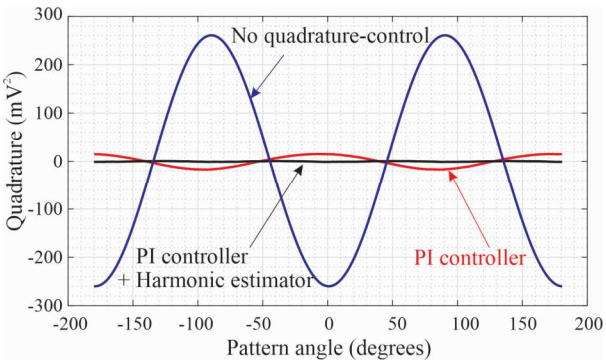

Figure 6: Variation of quadrature during the precession as a function of the pattern angle (θ). Effectiveness of the two control architectures for quadrature compensation is illustrated.

Similar to the conventional PI quadrature control, the output of the harmonic estimator is applied in-phase with the reference oscillation phase, therefore, the controller output does not interfere with precession [2].

Angular Gain Temperature Sensitivity

To study the correlation between Effective Angular Gain (EAG) and temperature, we first characterized the anisoelasticity of the DFP gyroscope at different temperatures. To perform this characterization, we utilized a Thorlabs TED-4015 temperature controller to locally control the temperature of the LCC package at different set-points with a variation within ±10 mK.

Initially, at 27 °C temperature, the frequency split ($\Delta\omega$) of the DFP gyroscope was electrostatically reduced to 20.3 mHz. To reduce the frequency split to 20.3 mHz, biasing voltages of 8.3 V and 3.009 V were applied along the X and Y axes, respectively.

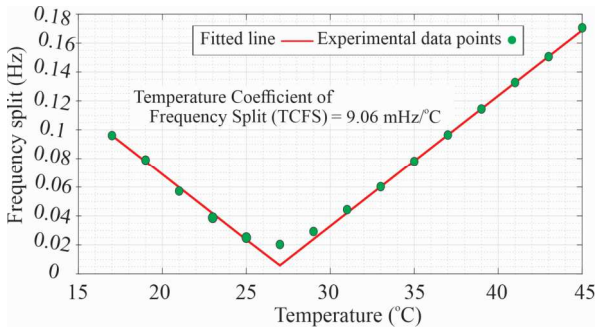

Figure 7: The frequency-split of DFP MRIG at different temperatures. The mismatch in TCF, between the two operational modes, was estimated to be on the order of 603 ppb/C.

After electrostatic frequency tuning, the DFP gyroscope was physically rotated at an input angular rate of 360 DPS and variation of the actuation frequency as a function of pattern angle was utilized to characterize the anisoelasticity of the DFP gyroscope. The experiment was repeated for different temperatures, ranging from 17 °C to 45 °C with 2 °C increments, with results shown in Fig. 7.

For the DFP gyroscope under test, the Temperature Coefficient of Frequency Split (TCFS) was estimated to be on the order of 9.06 mHz/C. These results indicate a TCF-mismatch on the order of 603 ppb, potentially caused by stresses of die-attachment and packaging [10] or the crystalline orientation-dependent TCF of the silicon material, [11].

Similarly, the EAG was characterized at different temperatures, shown in Fig. 8. The correlation between frequency-split and EAG variations can be noted from figures 7 and 8. By utilizing the harmonic estimator along with the PI controller, Fig. 2, a 63-times reduction in AGTS was obtained, as illustrated in Fig. 8. The improvement was attributed to effectiveness of the DQC architecture that fully eliminated ellipticity.

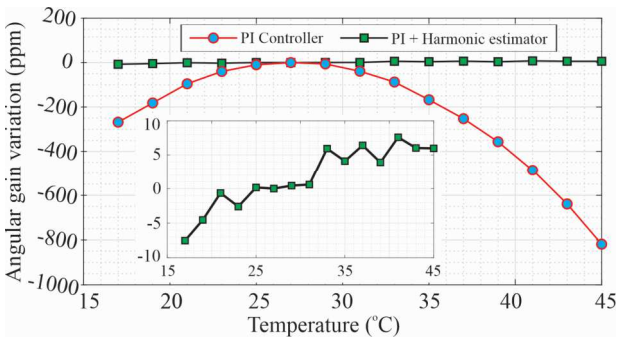

Figure 8: Experimental data on variations of angular gain as a function of temperature. It is demonstrated that by utilizing the DQC, the AGTS was reduced by ~63-times.

Angular Gain Stability

We tested the improvement in accuracy of direct angle measurements and angular gain stability. In the experiment, the DFP gyroscope was rotated at 360 DPS for 10 hours, and the temperature around the LCC was changed with a profile shown in Fig. 9(a). Based on the EAG at 27 °C, the time-linear variation in angle was subtracted from the output of the gyroscope and the angular drift was plotted as a function of time in Fig. 9(b).

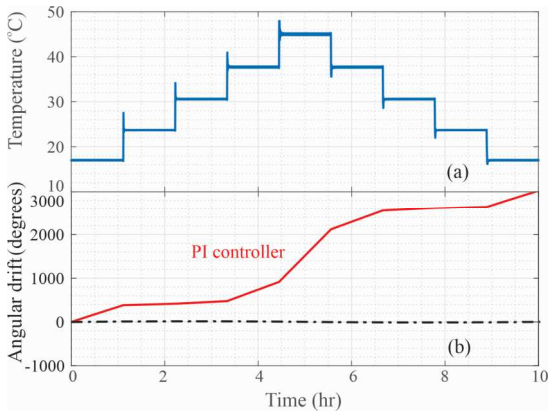

Figure 9: Angular drift of the DFP gyroscope which was rotated at 360 DPS and introduced to dynamic temperature changes, with a profile shown in (a), is demonstrated in (b).

As demonstrated in Fig. 9(b), in the case of using a PI quadrature control after 10 hours of operation, the angular drift was on the order of 3000 degrees. The slope of angular drift in each interval corresponded to variation of the EAG demonstrated in Fig. 8. By repeating the same experiment and applying the DQC, we were able to reduce the angular drift to less than 25 degrees.

To characterize stability of the EAG, we numerically calculated the time-derivative of the measured angle and applied the Allan variance characterization method. As illustrated in Fig. 10, we achieved 10-times improvement in EAG stability for operation during dynamic temperature changes from 17 °C to 45 °C by applying the DQC.

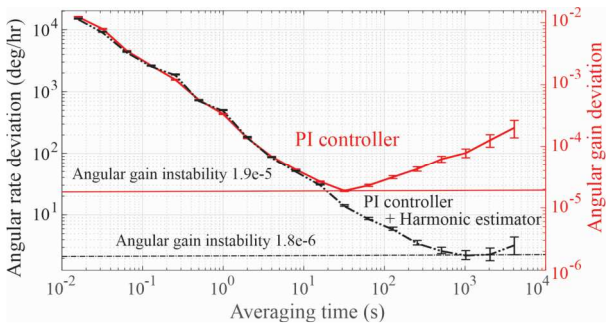

Figure 10: Allan variance of time-derivative of angle output. The results were used to characterize the stability of effective angular gain in operation through dynamic temperature fluctuations.

CONCLUSION

In this paper, we studied temperature sensitivity of angular gain in MRIG caused by variations in anisoelasticity. We demonstrated that the Angular-Gain-Temperature-Sensitivity (AGTS) is dependent on the effectiveness of the quadrature control, which is utilized to reduce ellipticity and compensate for anisoelasticity.

We experimentally demonstrated that by applying a harmonic estimator along with a proportional-integral feedback loop as the quadrature control, AGTS was reduced by 63-times. The reduction in AGTS, enabled us to measure sub-degree-per-hour angular rates at ambient temperature and angular rates on the order of 2.1 deg/hr when operating in an environment with dynamic temperature variations.

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